MACM 202 Assignment 2, Spring 2004

This assignment is worth 10% of your grade. It is due Thursday February 5th at 10am. A late penalty of 20% will apply for each day late. Do each question in a separate Maple worksheet and hand in a printout of each worksheet.

Question 1 (10 marks) Locate, using any method, a stable 3-cycle in the logistic map $f(x) = ax(1-x)$. Determine the range end points a_L and a_R to 10 decimal digits of precision such that for $a_L < a < a_R$ the 3-cycle is stable.

Question 2 (40 marks) For the period doubling bifurcations in the logistic map f(x) = $ax(1-x)$ that begin at $a = 3$ compute as many of the period doubling bifurcation points as you can. Estimate the value of Feigenbaum's constant from the values you obtain.

Question 3 (20 marks) Design a boolean network with four nodes which counts in binary from 0 to 15 and then cycles. You may use more than four nodes if you wish but you must identify the four nodes which form the counter. Run your network using the `run` procedure in the networks worksheet under the assignment lab directory and show the output.

Question 4 (20 marks) Consider a circular boolean network of $n \geq 3$ nodes numbered $1, 2, ..., n$, i.e. each node has one input. Suppose three of the nodes, say nodes $1, s, t$ where $1 < s < t \leq n$ apply the "Inverse" rule and the remaining $n - 3$ nodes apply the "Identity" rule. For "sufficiently many" values of (n, s, t) compute the length of the cycle resulting from the initial state $[0, 0, ..., 0]$. Use the `run2cycle` procedure in the networks worksheet. Study the data that you obtain and come up with a general rule, for the length of the cycle as a function of (n, s, t). Now verify your rule by running "sufficiently many" values for n, s, t.

Question 5 (10 marks) Write a Maple procedure called `ISPRIME` which on input of an integer n outputs true if n is prime and false otherwise. This is a simple programming exercise to help you learn to program in Maple. You should do this before Question 2. Use the simple method trial division, i.e. test if n is divisible by $2, 3, 5, 7, 9, 11, 13, 15, ..., \lfloor\sqrt{n}\rfloor$. Use the Maple command `irem` for integer division. Test your code on all odd numbers between 950 and 1000. Use the `isprime` command to check your answers.