MACM 202 Assignment 6, Spring 2004

Michael Monagan

This last assignment is due Tuesday April 6th (last day of class) at 10:30am. It is worth 10% of your final grade. The penalty for a late assignment is 20% for each day late.

Question 1: Linear First Order Systems

Do problems 5.6, 5.10 from the text. For problem 5.6, are there any positive values for K_1, K_2, K_3 that result in an oscillating solution?

Question 2: Non-Linear First Order Systems

Do problems 5.16, 5.17 from the text. For 5.16 part (c) use DEplot. Also give a physical interpretation for each term (or pair of terms) in the two differential equations. For 5.17 part (d) graph each type of behaviour found using DEplot.

Question 3: First Order Modeling Problem

On assignment 5 I asked you to either model a fish population with a seasonal harvest or to model the spread of a virus through a population. For assignment 6 do the question you didn't do on assignment 5.

Question 4: A Spring Modeling Problem

This exercise is to model one of the vibrations of CO_2, carbon dioxide. Let M_C be the mass of a carbon atom and M_O the mass of an oxygen atom. We will model a vibration of the bonds connecting the carbon atom to the oxygen atoms by connecting them with springs. Imagine that the atoms are in a line with the carbon atom C connected to each oxygen atom by an identical spring with spring constant k. Let d be the rest length of the spring, i.e., if the carbon atom is distance d from the oxygen atom, no force is applied by the spring to either atom.

Let $C(t)$ be the position of the carbon atom at time t and let $y(t)$ and $z(t)$ be the positions of the oxygen atoms at time t. Assume the rest position of the atoms are at $x = -d, 0, d$ for the oxygen, carbon and oxygen atoms respectively. Assuming Hooks' law, construct a system of three second order differential equations to model the motion of the three atoms assuming no friction. For

$$d = 2, C(0) = 0, C'(0) = 0, y(0) = -1.5d, y'(0) = 0, z(0) = 1.5d, z'(0) = 0$$

and suitable values for M_C, M_O, k, solve the equations using dsolve and graph the solutions. You should obtain $C(t) = 0$ and an oscillating solution for $y(t)$ and $z(t)$ if the differential equations are correct. Now, for

$$d = 2, C(0) = 0, C'(0) = 0, y(0) = -d, y'(0) = 0, z(0) = 1.5d, z'(0) = 0$$

and suitable values for M_C, M_O, k, solve the equations using dsolve and graph the solutions.

If anyone feels keen to work on this question a bit more, try to generate an animation of the three atoms placed on the horizontal axis. Represent each atom as a square and the position of each atom as a square, as a POLYGONS([....]).

Question 5: Partial Derivative Plots

Consider the function

$$f(x, y) = 1 - x^2 - y^3 - 2xy.$$

(a) Generate a 3-dimensional plot of $f(x, y)$ on $-2 < x < 2$ and $-2 < y < 2$ depicting also the lines $f(x, 1)$ and $f(1, y)$. See the `plots[spacecurve]` command for drawing a line in 3-dimensions.

When drawing the surface $f(x, y)$ using the `plot3d` command, use the options `axes=frame` and `style=patchcontour`. To make the curves $f(x, 1)$ and $f(1, y)$ more visible, use the `thickness=n` option and it may help to graph two curves for $f(x, 1)$, namely, $f(x, 1)+\epsilon$ and $f(x, 1) - \epsilon$ for small ϵ so that the curve is more clearly visible above and below the surface.

(b) To graphically depict the partial derivatives of f at the point $(x = 1, y = 1)$ include in your plot from part (a) the tangent lines at $(x = 1, y = 1)$. Visually check that the tangent lines are correct, i.e., they are tangent to the surface at the point $(1, 1, f(1, 1))$.

(c) Compute the critical points of $f(x, y)$, i.e., the solutions of $\{f_x(x, y) = 0, f_y(x, y) = 0\}$ using the `solve` command. Generate a 3-dimensional plot of $f(x, y)$ which shows visually the location of the critical points by drawing a vertical line segment through each critical point (see the `plottools` package). Visually identify whether each critical point is a local minimum, local maximum, saddle point or inflexion point.

The graph of $f(x, y)$ on $-2 < x < 2, -2 < y < 2$ has a fairly large vertical range. To identify the nature of the critical points you may want to restrict the vertical range to say $-3 < z < 3$. To do this, use the `view=[-2..2,-2..2,-3..3]` option to the `plots[display]` command.