MACM 202 Bonus Assignment, Fall 2005

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This is a bonus assignment worth up to 7% of your grade. The marks you obtain on this assignment will be added to the worst assignment mark that you obtained during the course. None of the questions are particularly difficult. You are to answer all questions in Maple - as practice for the final. It is due Monday December 19th in my office (K10501) at 9am. A late penalty of 10% will apply for each HOUR late.

Question 1: First Order Equations (10 marks)
Exercises 4.12 and 4.17 from the text.

Question 2: First Order Systems (15 marks)
Exercises 5.11 and 5.18 from the text. For 5.18 (d) use DEplot.

Question 3: The Kermack-McKendrick Epidemic Model (15 marks)
Read the article "What does Mathematics have to do with SARS" by Fred Brauer. On page 12 there is a plot of $S(t)$ verses $I(t)$. Using `dsolve(...,numeric)` and `plots[odeplot]`, and the given data $\beta = 0.0178, \gamma = 2.73, S(0) = 254, I(0) = 7$ and $N = 161$, reproduce the plot on page 12 and also a plot of $S(t), I(t), R(t)$ verses t. The author solves an equation to determine $S_\infty = 76$. Solve this equation using Maple.

Question 4: Direction Field Plots (30 marks)
Consider the first order system $x'(t) = f(x(t), y(t)), \quad y'(t) = g(x(t), y(t))$.
The Maple `DEplot` command draws the direction field as a grid of arrows. Write a Maple procedure FieldPlot that on input of two functions (procedures) $f(x, y)$ and $g(x, y)$ and values a, b, c, d, m, n outputs a PLOT of an m by n grid of arrows on $x = a..b$ and $y = c..d$. Each arrow should consist of two polygons, a rectangle and a triangle, that is, of the form

POLYGONS($[p, q, r, s]$, $[u, v, w]$, COLOR(RGB, 1, 0, 0), STYLE(PATCHNOGRID))

where p, q, r, s, a, b, c are points of the form $[x, y]$. Test your program on

```
> f := (x,y) -> 2*x-x^2-1*x*y;
> g := (x,y) -> 0.5*x*y-0.2*y;
> FieldPlot( f, g, 0, 3, -1, 2, 17, 25 );
```