MATH 152 Assignment 8, Fall 2022.

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Webassign Exercises

11.10 Exercises 62, 76.

11.11 Exercises 9, 13, 28

9.1 Exercise 5, 9, 14

9.3 Exercises 11, 38, 45

7.4 Exercise 47.

Written Exercises

1 Section 11.11. $T_4(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$ is the degree 4 Taylor polynomial for $f(x) = e^x$. It can be used to approximate e^x for small x.

(a) Calculate $e^{0.5}$ and $T_4(0.5)$ and $e^{0.125}$ and $T_4(0.125)$. What are the actual errors?

(b) Use Taylor's inequality on page 762 to bound the error of $T_4(0.5)$ and $T_4(0.125)$.

Notice that the error bound for $T_4(0.125)$ is a lot less than for $T_4(0.5)$. To exploit this we will use $T_4(0.125)$ and the identity $e^x = (e^{x/2})^2$ to approximate $e^{0.5}$ using

$$e^{0.5} = (e^{0.25})^2 = ((e^{0.125})^2)^2 = (e^{0.125})^4 \approx T_4(0.125)^4.$$

(c) Calculate $T_4(0.125)^4$. How many decimal places of accuracy do you get for $e^{0.5}$?

This is basically how your calculator computes e^x. It uses the identity $e^x = (e^{x/2})^2$ for large x and a Taylor polynomial $T_n(x)$ for small x.

2 Section 9.1. For what values of k does $y = e^{kt}$ satisfy the differential equation $2y'' + y' - 6y = 0$?

3 Section 9.1

(a) If $y'(x) = 2x$ and $y(0) = 1$ find $y(x)$.

(b) If $y'(t) = 2y(t)$ and $y(0) = 1$ find $y(t)$.

4 Section 4.9 exercise 65.

5 Section 9.3. Solve the initial value problem $y' = 2ty - 2t$ with $y(0) = 3$ for $y(t)$.

6 Section 7.4 exercise 41.

7 Use the tan half angle substitution $t = \tan\frac{x}{2}$ to calculate $\int \frac{dx}{1-\cos(x)}$ and $\int \frac{1+\sin x}{1+\cos x}dx$. See Section 7.4 exercise 59. After substitution you should end up with $\int \frac{1}{t^2}dt$ and $\int \frac{t^2+2t+1}{1+t^2}dt$ respectively. Now integrate these rational functions and to reverse the substitution use $\tan\frac{x}{2} = (1-\cos x)/\sin x$ to express the final answer in terms of $\sin x$ and $\cos x$.

The Final Exam is Monday December 12th at 8:30am in B9200 and B9201.
Is will you some info about the final exam on Monday December 5th, the last day of class.