Let $A^{(0)} = A$ and $A^{(k)}$ be the matrix after the k'th step

Ordinary Gaussian elimination

$$R_i \leftarrow R_i - \frac{A_{ik}^{(k-1)}}{A_{kk}^{(k-1)}} \cdot R_k$$

for $k = 1$ to $n-1$ (step k)
 for $i = k+1$ to n (row i)
  for $j = k+1$ to n

$$A_{ij}^{(k)} := A_{ij}^{(k-1)} - \frac{A_{ik}^{(k-1)}}{A_{kk}^{(k-1)}} \cdot A_{kj}^{(k-1)}$$

$$\begin{array}{c|c} & \begin{matrix} & k & & j \end{matrix} \\ \begin{matrix} \\ k \\ \\ i \end{matrix} & \begin{bmatrix} 4 & \times & & \times \\ 0 & \boxed{3} & & 2 \\ 0 & \times & & \times \\ 0 & \boxed{5} & & 11 \end{bmatrix} \end{array}$$

$$= 11 - \frac{5}{3} \cdot 2 = \frac{23}{3}$$

Clear fractions

$$R_i \leftarrow A_{kk}^{(k-1)} R_i - A_{ik}^{(k-1)} R_j$$

$$A_{ij}^{(k)} := A_{kk}^{(k-1)} A_{ij}^{(k-1)} - A_{ik}^{(k-1)} A_{kj}^{(k-1)}$$

$$\det(A^{(k)}) = \left(A_{kk}^{(k-1)}\right)^{n-k} \det(A^{(k-1)})$$

Bareiss/Edmonds: $A_{00}^{(-1)} := 1$

$$A_{ij}^{(k)} := \frac{A_{kk}^{(k-1)} A_{ij}^{(k-1)} - A_{ik}^{(k-1)} A_{kj}^{(k-1)}}{A_{k-2\,k-2}^{(k-2)}}$$

current pivot

previous pivot

Theorem (Jack Edmonds 1967, Erwin Bareiss 1968, Jordan 1838–1922)
The division by $A_{k-2\,k-2}^{(k-2)}$ is exact in R and $\det(A) = A_{nn}^{(n-1)}$

Moreover $A_{kk}^{(k-1)} = \det(k \times k$ principle minor of $A)$ and

$$A_{ij}^{(k-1)} = \det(k \times k \text{ minor of } A)$$
$i \geq k, j \geq k$