THE REAL WORLD, MATHEMATICS AND FRIDAY THE 13TH

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Typeset by $\mathcal{AMS}$-TEX

Preamble

• We live, both philosophically and technologically, in a mathematical world. More so than most of us care to admit.

• Our faith in science is in large part based on the "unreasonable efficacy of mathematics" that has been apparent at least since the time of Newton.

• Mathematics has also been arcane and inaccessible.

"The science of pure mathematics ... may claim to be the most original creation of the human spirit"

Whitehead, Alfred North

"[Mathematics] is an independent world created out of pure intelligence."

Wordsworth, William (1770 - 1850)

2. The Main Theorem

Let $\mathcal{S}$ denote the analytic functions f on the open unit disk D that satisfy

$$|f(z)| \leq \frac{1}{(1-|z|)}\,, \qquad z \in D\,.$$

Theorem 1. *For $f \in \mathcal{S}$ with $f(0) = 1$. If λ is any arc of the circle of length ϵ,*

$$D \int_\lambda \log_+ |f(z)| d\mu(z) + \int_\lambda \log_- |f(z)| d\mu(z)$$

$$\geq C.$$

and, for any $p > 0$,

$$\int_\lambda |f(z)|^p d\mu(z) > E\epsilon \exp(-pF\epsilon^{-1}).$$

Here C, D, $E > 0$ and F are absolute constants.

15 Minutes of History

1700 B.C.E.

Ahmes (c. 1650 B.C.E.)

600 B.C.E.

Thales of Miletus (c. 630-c 550) ***
Pythagoras of Samos (c. 570-c. 490)

400 B.C.E.

Plato (427-347)
Eudoxus of Cnidos (c. 400-c. 347) ***
Aristotle (384-322)

300 B.C.E.

Euclid (fl. c. 295)
Archimedes of Syracuse (287-212) *****

100 C.E.

Ptolemy (Claudius Ptolemaeus) (c. 100-c. 170)

1100

Leonardo Fibonacci of Pisa (C. 1170-post 1240)

1400

Piero della Francesca (c. 1410-1492)
Leonardo da Vinci (1452-1519)
Scipione del Ferro (1465-1526)
Albrecht Dürer (1471-1528) ***
Nicolas Copernicus (1473-1543)

1500

Niccolò Fonntana (Tartaglia) (c. 1500-1557)
Girolamo Cardano (1501-1576) ***
Robert Recorde (1510-1558)

1600

René du Perron Descartes (1596-1650) ***
Pierre de Fermat (1601-1665)
Blaise Pascal (1623-1662)
Isaac Newton (1642-1727) *****
Gottfried Wilhelm Leibniz (1646-1716)
Jacques Bernoulli (James, Jakob) (1654-1705)

1700

Leonhard Euler (1707-1783) ****
Joseph Louis Lagrange (1736-1813)
Pierre Simon de Laplace (1749-1827)
Adrien-Marie Legendre (1752-1833)

1800

Carl Friedrich Gauss (1777-1855) *****
Augustin-Louis Cauchy (1789-1857)
Niels Henrik Abel (1802-1829)
Carl Gustav Jacob Jacobi (1804-1851)
Joseph Liouville (1809-1882)

1800 (continued)

Evariste Galois (1811-1832) ***
Charles Hermite (1822-1901)
Georg Friedrich Bernhard Riemann (1826-1866)
Jules Henri Poincaŕe (1854-1912)
David Hilbert (1862-1943) ***

1900

Bertrand Russell (1872-1970)
Albert Einstein (1879-1955) ***
John von Neumann (1903-1957)
Kurt Friedrich Gödel (1906-1978) ***
Alan Mathison Turing (1913-1954) ***

Why Mathematics Works.

• The structure is right. It has been for a very long time.

• Axioms (assumptions).

• Theorems (facts that follow the axioms by rules).

• Rules (like the excluded middle).

• Change the rules and invisible unicorns exist.

• Russel's paradox.

"In mathematics you don't understand things. You just get used to them."

von Neumann, Johann (1903 - 1957)

Why Mathematicians Work

Compulsive Personality Disorder

According to DSM–III must meet 4 of the following:

• restricted ability to express warmth ... unduly conventional, serious and formal

• perfectionism (that misses the forest for the trees)

• insistence that others submit to his or her way of doing things

• excessive devotion to work and productivity

• indecisiveness: decision-making is either avoided, postponed or protracted

What It Works For.

- Weather (Chaos)
- Lotteries (Chance) and beating Lotteries
- Missiles (and missile failure)
- Putting neurosurgeons out of work
- Scheduling airlines
- Picking up garbage
- Understanding the unfairness of life (and elections)

"Music is the pleasure the human soul experiences from counting without being aware that it is counting."

Leibniz, Gottfried Whilhem (1646-1716

The Limits of Mathematics.

- Goedel and Incompleteness
- Turing and Computability
- Chaos (Weather)

A mathematician of the first rank, Laplace quickly revealed himself as only a mediocre administrator; from his first work we saw that we had been deceived. Laplace saw no question from its true point of view; he sought subtleties everywhere; had only doubtful ideas, and finally carried the spirit of the infinitely small into administration.

Napoleon (1769-1821)

The Philosopy of Mathematics.

- Formalists
- Platonists
- Constructivists

"Numbers are intellectual witnesses that belong only to mankind."

Balzac, Honore de (1799 - 1850)

"In my opinion, a mathematician, in so far as he is a mathematician, need not preoccupy himself with philosophy – an opinion, moreover, which has been expressed by many philosophers."

Lebesgue, Henri (1875 - 1941)

"The mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful."

Aristotle (ca 330 BC)

"Medicine makes people ill, mathematics make them sad and theology makes them sinful."

Luther, Martin (1483-1546)

"Anyone who cannot cope with mathematics is not fully human. At best he is a tolerable subhuman who has learned to wear shoes, bathe, and not make messes in the house."

Heinlein, Robert A.