

1 Sparse GCD Problems

$$g = \text{GCD}(a, b), \quad ca = a/g, \quad cb = b/g$$

Fully sparse: $g, ac, bc = c_1x_1^{50} + c_2x_2^{50} + \dots + c_nx_n^{50} + c_px_1x_2\dots x_n + c_0$

# vars	expand	divide	gcdheu	gcdeh	linzip
2	0.00	0.00	0.12	0.01	0.05
4	0.00	0.00	fail	0.02	0.08
8	0.00	0.00	fail	0.04	0.33
12	0.00	0.00	fail	0.09	0.83
16	0.00	0.00	fail	0.14	1.71
20	0.00	0.00	fail	0.21	3.11
24	0.00	0.00	fail	0.28	5.24

Sparse w/coeff: $g, ac, bc = c_1x_1^{50}x_2 + c_2x_2^{50}x_3 + \dots + c_nx_n^{50}x_1 + c_px_1x_2\dots x_n + c_0$

# vars	expand	divide	gcdheu	gcdeh	linzip
2	0.00	0.00	0.09	0.77	0.05
4	0.00	0.00	fail	2.54	0.07
8	0.00	0.00	fail	9.11	0.40
12	0.00	0.00	fail	22.93	1.20
16	0.00	0.00	fail	82.80	2.83
20	0.00	0.00	fail	960.15	5.34
22	0.00	0.00	fail	3444.42	7.41

Sparse content: $g, ac, bc = (c_1x_1^{100} + \dots + c_jx_j^{100} + c_{01})(c_{j+1}x_{j+1}^{100} + \dots + c_nx_n^{100} + c_{02})$

# vars	expand	divide	gcdheu	gcdeh	linzip
4	0.01	0.00	fail	1.51	0.33
6	0.01	0.00	fail	8.02	0.24
8	0.01	0.01	fail	10.96	0.56
10	0.02	0.05	fail	21.85	0.96
12	0.03	0.08	fail	32.39	1.70
14	0.04	0.11	fail	38.82	2.76
16	0.07	0.17	fail	59.68	4.74

2 Dense GCD Problems

Dense: $g, ac, bc = c_{0,0,\dots,0} + c_{1,0,\dots,0}x_1 + \dots + c_{0,0,\dots,1}x_n + \dots + c_{d,0,\dots,0}x_1^d + c_{d-1,1,\dots,0}x_1^{d-1}x_2 + \dots + c_{0,0,\dots,d}x_n^d$

3 variables ($\mathbf{Z}[x, y, z]$):

degree	expand	divide	gcdheu	gcdeh	linzip
6	0.01	0.01	0.04	0.10	0.06
8	0.05	0.06	0.09	0.30	0.06
10	0.18	0.16	0.34	0.85	0.27
12	0.53	0.39	0.84	2.34	0.59
14	1.17	0.83	1.90	5.76	1.23
16	2.94	1.64	4.21	12.42	2.45
18	5.85	3.01	8.65	25.13	4.52
20	12.82	5.29	16.79	19.59	8.16
22	23.17	8.96	29.42	89.34	14.28
24	38.87	14.61	49.61	156.07	23.81
26	75.81	23.22	80.60	278.34	38.88
28	123.66	36.22	132.07	475.05	61.99
30	193.24	57.51	207.98	766.68	98.78

4 variables ($\mathbf{Z}[w, x, y, z]$):

Degree	expand	divide	gcdheu	gcdeh	linzip
4	0.01	0.01	0.04	0.08	0.05
6	0.14	0.10	0.75	0.54	0.26
8	0.76	0.57	9.66	2.06	1.24
10	3.74	2.63	59.60	13.09	5.42
12	17.28	10.38	275.68	32.16	21.45
14	57.12	35.56	1241.82	247.16	72.91

3 Semi-dense GCD Problems

Similar to dense, but total number of terms is the square root of the maximum.

3 variables ($\mathbf{Z}[x, y, z]$):

Degree	Expand	Divide	gcdheu	gcdeh	linzip
10	0.00	0.00	42.34	0.01	0.01
15	0.00	0.00	1693.66	0.02	0.03
20	0.00	0.00	fail	0.03	0.03
35	0.00	0.00	fail	0.10	0.06
50	0.00	0.00	fail	0.25	0.09
65	0.02	0.01	fail	0.51	0.12
80	0.01	0.01	fail	0.98	0.18
95	0.03	0.02	fail	39.36	0.21
110	0.04	0.02	fail	3.26	0.29
140	0.05	0.03	fail	5.78	0.44
170	0.07	0.06	fail	13.60	0.60
200	0.09	0.09	fail	9.56	0.85

4 variables ($\mathbf{Z}[w, x, y, z]$):

Degree	Expand	Divide	gcdheu	gcdeh	linzip
5	0.00	0.00	16.83	0.01	0.03
10	0.00	0.00	fail	0.02	0.02
15	0.00	0.00	fail	0.03	0.03
30	0.00	0.00	fail	0.08	0.07
45	0.00	0.00	fail	0.20	0.11
60	0.01	0.01	fail	0.49	0.16
75	0.01	0.03	fail	0.89	0.22
90	0.03	0.02	fail	1.60	0.30
105	0.02	0.02	fail	2.58	0.40
120	0.02	0.02	fail	3.73	0.50
135	0.03	0.03	fail	3.18	0.62
150	0.04	0.06	fail	7.82	0.76