# MACM 204 Final Exam
# Fall 2019

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Do not open this booklet until told to do so.

You may have bring a one page cheat sheet (8.5" x 11") (two sided) into the exam.
No other notes or textbooks are permitted.
You may not access the internet or use any electronic device except Maple.
There are five questions worth 80 marks. Attempt all questions.
**Use Maple to answer all questions.**
Time allowed 2.5 hours (150 minutes).

**Answer each question in a separate Maple worksheet.**

**Write your name, student ID and question number at the top of each worksheet.**

Save each worksheet in a separate file on your computer.

When you are finished you must print your solutions.

You may write on the back of this exam.
Please hand it in with your solutions.

## Question 1 (25 marks)

**Part (a)** Let $f(x) = x^4 - 9$ and $g(x) = 2 \cdot x \cdot (3 - x)$.
Graph $f(x)$ and $g(x)$ on the domain $-3 \le x \le 3$.

**Part (b)** Solve $f(x) = g(x)$ for $x$ and calculate the solution near $x = 2.0$ to 10 decimal places.

**Part (c)** Let $A$ be the area between $f(x)$ and $g(x)$. Calculate $A$ to 5 decimal places.

**Part (d)** Calculate the maximum of $h(x) = x^2 \cdot e^{-x}$ for $x \ge 0$.

**Part (e)** Evaluate and simplify $\sum_{i=1}^{n} i^2 \binom{n}{i}$ .

**Part (f)** Solve the differential equation $\frac{dy}{dx} = \left(4 - \frac{y}{5}\right)$ in Maple with the initial value $y(0) = 40$.

**Part (g)** Recall that the degree $n$ Taylor polynomial for $\ln(1+x)$ is

$$T_n(x) = \sum_{i=1}^{n} \frac{(-1)^{(i-1)} \cdot x^i}{i}.$$

For example, $T_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3}$. Calculate $T_n(x)$ for $1 \le n \le 5$.

**Part (h)** Consider the identity $\tan\left(\frac{x}{2}\right) = \frac{(1 - \cos(x))}{\sin(x)}$. Use Maple to show that it is true or very probably true using two different approaches.

# Question 2 (13 marks)

**Save your work from the previous question!**
Open a new worksheet for this question

Consider the definite integral $A = \int_a^b f(x)\,\mathrm{d}x$. Recall that Simpsons rule

$$Sn = \frac{h}{3}\cdot[f(a) + 4\cdot f(a+h) + 2\cdot f(a+2\cdot h) + 4\cdot f(a+3\cdot h) + ... + 2\cdot f(a+(n-1)\cdot h) + f(a+n\cdot h)]$$

is an approximation of A. Here $n$ is the number of sub-intervals and $h = \frac{(b-a)}{n}$ is the width of each subinterval and $n$ has to be even.
Write a Maple procedure **Simpson** such that **Simpson**$(f, x, a, b, n)$ outputs $Sn$
For example, you should get

```
> f := sin(x);
```

$$f := \sin(x) \tag{2.1}$$

```
> Simpson(f,x,0,Pi,4);
```

$$2.004559755 \tag{2.2}$$

```
> Simpson(f,x,0,Pi,16);
```

$$2.000016590 \tag{2.3}$$

Test your procedure on those two inputs and also on

```
> Simpson(f,x,0,Pi,3);
Error, (in Simpson) n must be positive and even
```

## Question 3 (15 marks)

Consider the following function $f(x, y)$.

```
> f := 12-x^2-2*y^2-x^3-y-y^3;
```

$$f := -x^3 - y^3 - x^2 - 2\,y^2 - y + 12 \tag{3.1}$$

**Part (a)** Calculate the partial derivatives $f_x(x, y)$ and $f_y(x, y)$ using Maple's **diff** command.

**Part (b)** Solve the system $f_x(x, y) = 0$ **and** $f_y(x, y) = 0$ to find the critical points of $f(x, y)$.

**Part (c)** Determine the Tangent plane $z = T(x, y)$ for $f(x, y)$ at the point $x = -1,\ y = 1$.

**Part (d)** Create a 3 dimensional plot of $f(x, y)$ and $T(x, y)$ on a suitable the domain using suitable options.

**Part (e)** The tangent lines are the lines $z = T(x, 1)$ and $z = T(-1, y)$ in $\mathbb{R}^3$. They can also be obtained using $T(x, 1) = f(-1, 1) + f_x(-1, 1)\cdot(x+1)$ and $T(-1, y) = f_y(-1, 1)\cdot(y-1)$. Using the **spacecurve** command in the plots package, graph $T(x, 1)$ for $-2 \le x \le 1$ in red and $T(-1, y)$ for $0 \le y \le 2$ in blue and display them together with $f(x, y)$ on the same plot.

## Question 4 (13 marks)

Run the following random walk experiment.
Consider a unit square with corners $A = [0, 0]$, $B = [1, 0]$, $C = [1, 1]$, $D = [0, 1]$.

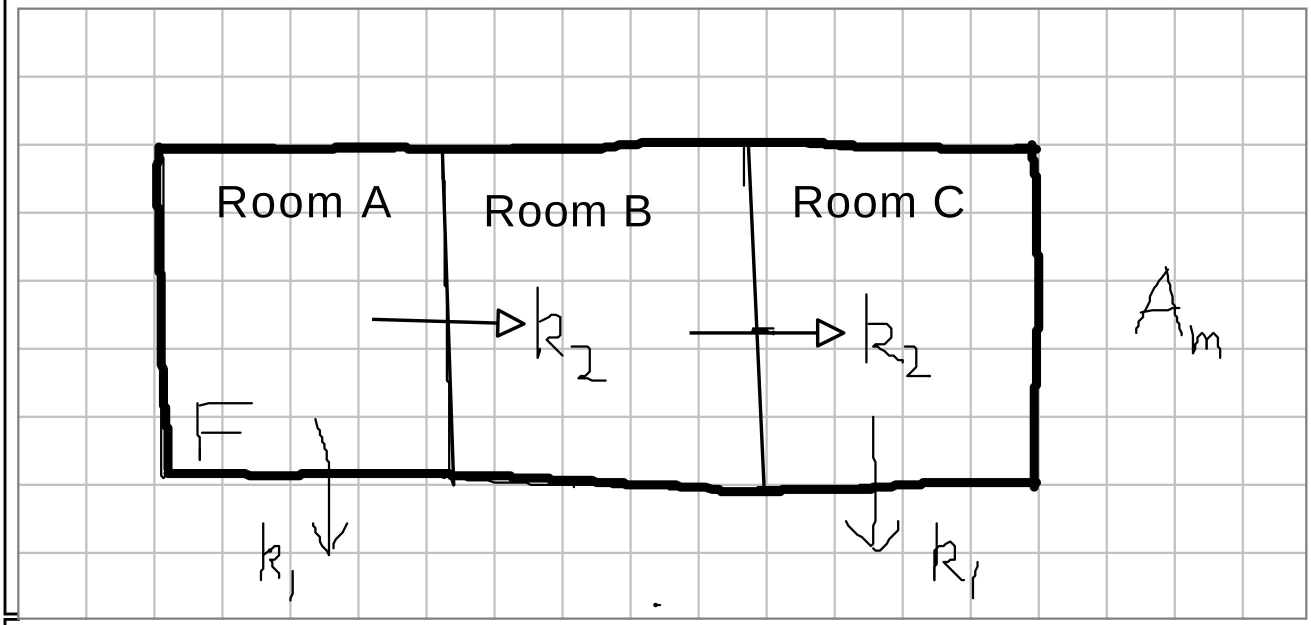
Starting at $X = [0.5, 0.5]$, repeat the following $N$ times.

Pick one of A, B, C and D at random.
If A is picked go half way (from the current position) to A.
If B is picked go two thirds of the way to B.
If C is picked go half way to C.
If D is picked go two thirds of the way to D.

Run the experiment in Maple for $N = 500$ steps and plot the sequence of points on the walk. You should get a fractal image that has 4 square shapes.

# Question 5 (14 marks)

Consider a house with three rooms $A, B, C$, a furnace $F$ in room $A$ and outside temperature $A_m$ degrees as drawn in the figure below. Here $k_1, k_2$ and $F$ are positive constants and $A_m$ is constant.




Let $A(t), B(t), C(t)$ be the temperature at time $t$ in rooms $A$, $B$, $C$ respectively.

**Part (a)** Using Newton's law of cooling to model the rate at which heat moves from room to room and to the outside , write down differential equations for $A'(t) = \ldots,\ B'(t) = \ldots,\ C'(t) = \ldots$ in Maple.

**Part (b)** Solve for the temperature equilibrium point as a function of $k_1, k_2, T, F$, that is, when $A'(t) = 0, B'(t) = 0, C'(t) = 0$. Try to express the formulas in a nice form. Do not solve the differential equations.

**Part (c)** Consider the following system of differential equations.

$$x'(t) = 2\,x(t) - 0.5\,x(t)\,y(t),\quad y'(t) = 0.2\,x(t)\,y(t) - 0.2\,y(t)$$

Using the **DEplot** command in the DEtools package, generate a field plot together with solutions curves for inital values $x(0) = 1, y(0) = 1$ and $x(0) = 3, y(0) = 3$, for the domain $0 \le t \le 20$.