

New options to visualize system of differential equations in Maple

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Abstract

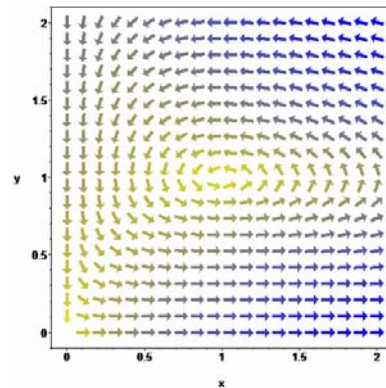
We present new options created for the *DEplot* command in Maple for enhancing the visualization of the direction fields in Maple and for animating the direction fields and the solution curves with respect to time. We also present a new Maplet, *DEplotlet* (*DEplotInteractive*), which is used to ease the use of the *DEplot* command, and a database of different systems of differential equations with parameters and initial values which can be selected for display and easily modified by the instructor.

One of the main usages of the *DEplot* command in Maple is to create a direction field of a differential equation or system of differential equations.

Following is a list of some of the new options which have been added for the *DEplot* command:

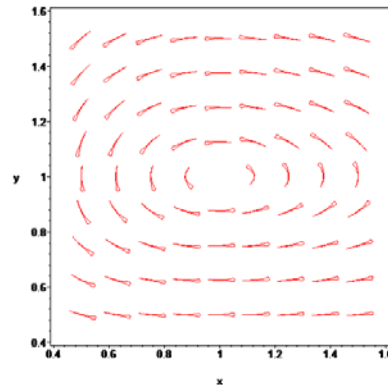
Velocity options:

color = velocity[yellow, blue]: the color of the arrows is determined by the velocity of the field. The low and high velocity colors are specified as the index of *velocity*. The option *color=velocity* can be use with the default of *[green, red]*.



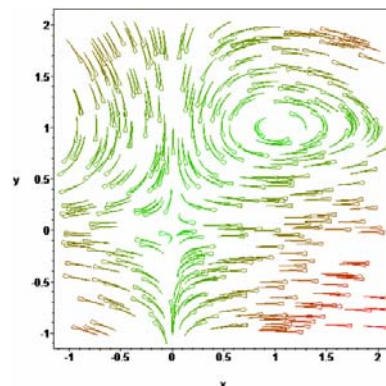
New arrows:

arrows=fish: This option creates a field of objects (fish shape) for which the curvature of each fish approximates the solution curve to improve the direction field. The curvature of each fish is approximated by the modified Euler's method.



dirfield options:

dirfield = 400: this option specifies the number of randomly positioned arrows in the direction field. 2 is the minimum. We have found that random placement of arrows is visually superior to a grid placement of arrows.

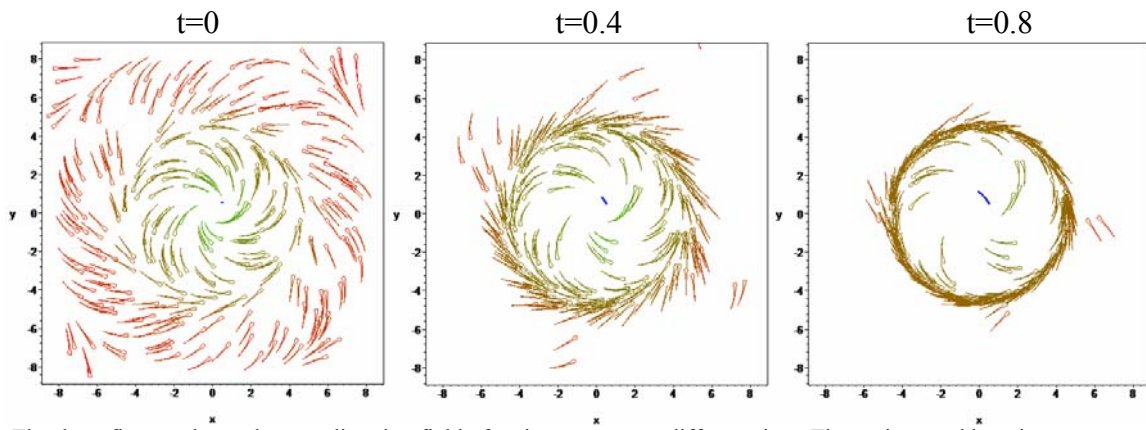


Animation options:

1- *animatefield* = true or false (default = false): *animatefield=true* will create an animation of the direction field with respect to time (i.e. the arrows describing the direction field move as a solution with respect to time). The default number of animation frames is 25, which can be changed with the *numframes* option. *dsolve [numeric]* is used to compute the trajectory of each arrow.

2- *animatecurves* = true or false (default = false): *animatecurves=true* will create an animation of the evolving solution curve from the specified initial value(s) with respect to time. If *animatefield* is not in use, the number of frames is obtained from *numsteps* (default of 50), but this can be overridden with the *numframes* option. *dsolve [numeric]* is used to compute the solution curves for all specified initial values at different times.

3- *animate* = true or false (default = false): *animate=true* is simply a shortcut that can be used to specify both *animatefield* (if applicable) and *animatecurves* (if applicable) be used.



The three figures above shows a direction field of a given system at different time. The option used here is *animate=true*.

In addition to the new options for the *DEplot* command we have created a Maplet which can read and write systems from and into a text editable library (database) and produce the direction field for the chosen system. Aside from plotting, the user can change the values of parameters in the system as well as few options in *DEplot* command. i.e arrows, range of variables. The editable database is designed so that it is a valid Maple input that is a plain text file, so that the user can read and edit the database easily. The idea of creating the database is to help educators browse different behaviors of a system of differential equations with different parameter settings and different initial values instantly. This will lead to the idea of having Maple databases of systems, functions and perhaps plots for other commands other than *DEplot* for educators. The database is an alternative way of storing examples than the examples in the Maple's help pages. We think that such databases for other Maple command and packages should be developed in the future.

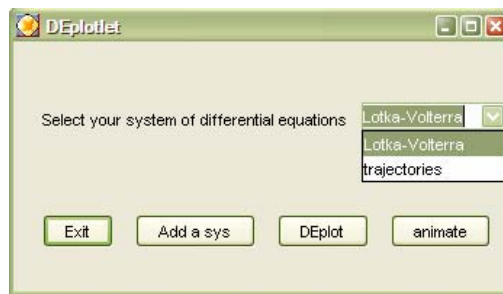
Below is a sample of the library file and the designed Maplet:

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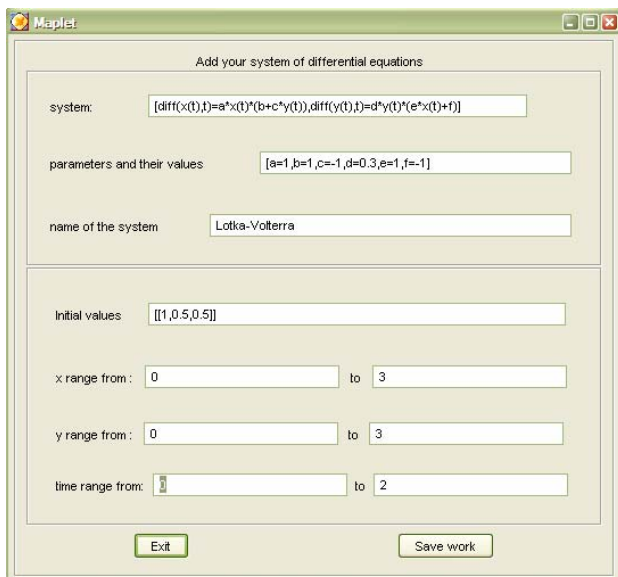
systems - Notepad
File Edit Format View Help
Lotka-volterra
sys=[diff(x(t),t) = a*x(t)*(b+c*y(t)), diff(y(t),t) = d*y(t)*(e*x(t)+f)];
par=[a = 1 , b = 1, c = -1, d = 0.3, e = 1, f = -1];
ini=[[1, 0.5, 0.5],[0,1.2,1.1]];
opt=[t=0..5,x=0..2,y=0..2];
trajectories
sys=[diff(x(t),t) = a*y(t)+b*x(t)+c, diff(y(t),t) = d*x(t)^2+e*y(t)+f];
par=[a = 1, b = -1, c = -2, d = 1, e = -1, f = 0];
ini=[[0, .5, .5],[0,6,0]];
opt=[t=0..2,x=-6..8,y=-6..8];

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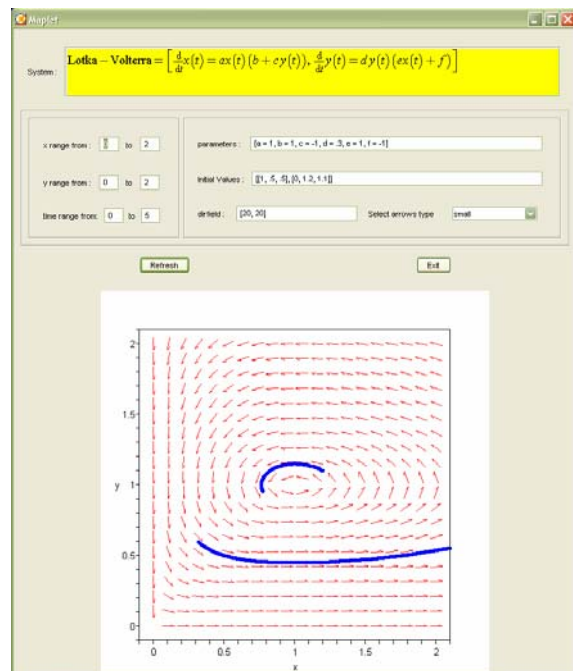
This is a Sample of the editable database file. The first line indicates the name of the system. The second line is a list of differential equations. The third line is the list of all parameters used in the system and their values. The fourth line is the list of initial values to be used, and the fifth line is the range of the variables time, x, y.



This is the main *DEplotlet* maplet which user can chose one of the systems read from the library to *DEplot* (or *animate* it). In addition, the user can chose to add a new system to the library.



This is the maplet for adding the system with parameters to the database. The name of the system as well as other information about the system can be specified.



Above is a sample of the *DEplotlet* page. The top window shows the name of the system and the system in a *Math-ML* view. The two windows below the *Math-ML* view are for different options of *DEplot* and also values of the parameters inside the system.