

**Due Date:** Wednesday September 30, 2020, 11:59pm

## Instructions

Answer all questions on paper or a tablet using your own handwriting. **Please number each page** Include a cover page with your name, student ID number and a list of the questions you have answered.

### Textbook Reading

- Sections: 11.1, 11.2, 3.4, 3.5, 3.6

### Definitions, Concepts & Keywords

- Understand more terminology for graphs and trees.
- Know when two paths, cycles, graphs, and trees are not the same.
- Can calculate basic probabilities for discrete events.

### Exercises

#### A. Textbook Questions

Section 11.1 Exercises 5.  
Section 11.2 Exercises 9(a), 12(b).  
Section 3.4 Exercises 6, 9.  
Section 3.5 Exercises 10 (omit  $Pr(A \triangle B)$ ).  
Section 3.6 Exercises 2, 8.

#### B. Instructors Questions

- For a positive integer  $n$ , what is the value of the counter after the following code has been executed. (Both C and Python code are included, use the language you are familiar with.)

C code:

```
int i, j, k, counter;
counter = 0;
for( i=1; i<=n; i++ )
    for( j=1; j<=i; j++ )
        for( k=1; k<=j; k++ )
            counter ++;
```

python code:

```
counter = 0
for i in range(1, n+1):
    for j in range(1, i+1):
        for k in range(1, j+1):
            counter = counter + 1
```

- Consider the graph  $G$  with vertices  $V = \{1, 2, 3, 4, 5\}$  and edges  $E = \{\{1, 2\}, \{2, 3\}, \{3, 1\}, \{3, 4\}, \{4, 5\}, \{5, 1\}\}$ .
  - Draw  $G$ .
  - How many cycles does  $G$  have? List them.
  - Is  $G$  bipartite? Explain.
  - Give a spanning subgraph of  $G$  that is a tree.
  - Draw the subgraph of  $G$  induced by the vertices  $\{2, 4, 5\}$ .
- A graph  $G = (V, E)$  is called regular if every vertex in  $G$  has the same degree. Draw a graph with 6 vertices where
  - every vertex has degree 1
  - every vertex has degree 2
  - every vertex has degree 3
  - every vertex has degree 4
- Up to isomorphism, find all unlabelled graphs on 5 vertices with 5 edges.

5. Let  $n$  and  $k$  be integers with  $3 \leq k \leq n$ .  
How many cycles of length  $k$  are there in the complete graph  $K_n$ ?  
Hint: we can select a cycle of length  $k$  from  $K_n$  by choosing a sequence of  $k$  distinct vertices  $v_1, v_2, \dots, v_k$  and then selecting the cycle with vertex set  $\{v_1, \dots, v_k\}$  and edge set  $\{\{v_1, v_2\}, \dots, \{v_{k-1}, v_k\}, \{v_k, v_1\}\}$ . It will help to determine (to avoid over-counting) for each cycle of length  $k$  how many sequences select this cycle.
6. How many subgraphs of  $K_n$  are isomorphic to  $K_{2,5}$ ?  
How many subgraphs of  $K_{n,n}$  are isomorphic to  $K_{2,5}$ ?
7. Let  $K_5^-$  be a graph obtained from  $K_5$  by deleting one edge.  
How many subgraphs of  $K_n$  are isomorphic to  $K_5^-$ ?
8. We say that two distinct edges in a graph are **adjacent** if there is a vertex incident to both (the two edges share exactly one vertex.) For the graph  $K_n$ , determine the number of sets of two edges  $\{e, f\}$  with the property that  $e, f$  are
  - (a) adjacent and
  - (b) not adjacent.
9. Suppose 6 men and 6 women attend a party.  
The host wants to select some of the 12 guests to play a game.
  - (a) If the host chooses two guests at random, what is the probability of choosing two women?
  - (a) If the host chooses two guests at random, what is the probability of choosing one man and one woman?
  - (c) If the host chooses four guests at random, what is the probability of choosing two women and two men? You should get  $5/11$ .
10. At a university there are 300 first year computing majors. Suppose 100 can program in Python, 60 can program in Java, 80 can program in C++, 20 can program in Python and Java, 20 can program in Java and C++, 10 can program in Python and C++ and none can program in all three languages.
  - (a) If a student is selected at random, what is the probability that they can program in exactly 2 languages.
  - (b) If a student is selected at random, what is the probability that they can only program in Python?
  - (c) If two students are picked at random, what is the probability they can both program in Java?  
Hint: use a Venn diagram.
11. Let  $S$  be a finite sample space and  $A, B$  and  $C$  be subsets of  $S$ .
  - (a) In class we showed that  

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B).$$
 Generalize this to find a formula for  $Pr(A \cup B \cup C)$ .
  - (b) Let set  $S = \{1, 2, \dots, 60\}$ . Using the formula from part (a), find the probability of choosing  $x \in S$  that is divisible by 2 **or** 3 **or** 5. You should get  $11/15$ .
12. A US company has developed an antibody test for whether a person has had COVID. If a person has had COVID, the probability that the test is positive is 0.98. If a person has NOT had COVID, the probability that the test is positive is 0.03. Suppose 10% of Americans have had COVID – which will probably be true by the time you are reading this. Use Bayes' theorem to determine the probability that an American who tests positive has had COVID.