

# Lecture 14 Solving first order non-homogeneous RRs

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Grimaldi 10.3



Can you move the disks on pole 1 to pole 3 using pole 2 as needed?

Rule 1: move one disk at a time.

Rule 2: do not put a disk on top of a smaller disk.

Question: how many moves is necessary?

# Solving first order non-homogeneous recurrences

Consider the non-homogenous recurrence relations

(1)  $a_n + c_1 a_{n-1} = f(n)$  where  $c_1 \neq 0$  and  $f(n) \neq 0$

(2)  $x_n + c_1 x_{n-1} + c_2 x_{n-2} = f(n)$  where  $c_2 \neq 0$  and  $f(n) \neq 0$

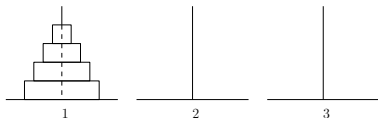
How do we solve them?

Case (1)  $a_n + c_1 a_{n-1} = f(n)$  where  $c_1 = -1$ .

Example 1. Solve  $a_n - a_{n-1} = 3n^2$  with  $a_0 = 7$ .

Example 2. Solve  $a_n - 3a_{n-1} = 5 \cdot 3^n$  with  $a_0 = 2$ .

### Example 3 – The Towers of Hanoi.



Move the disks from pole 1 to pole 3 using pole 2 as needed.

Move one disk at a time. Do not put a bigger disk on top of a smaller one.

(1)

(2)

(3)

Let  $m_n$  be the **number of moves**.

Determine and solve a recurrence relation for  $m_n$ .

## Example 3 – The Towers of Hanoi (cont.)

Example 4 – Interest on a loan.

Pauline takes out a bank loan for  $\$S$  dollars. She pays back  $\$P$  every month and the bank charges her  $r\%$  interest per month. Let  $a_n$  be the amount she owes after  $n$  months. Determine, and solve, a recurrence relation for  $a_n$ .

## Example 4 – Interest on a loan (cont.)