

Lecture 4: Combinations with repetition: Grimaldi 1.4

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How many combinations of size 3 are there from $S = \{a, b, c\}$ if repetitions are allowed?

Theorem (combinations with repetitions)

Let S be a set with n elements. The number of ways to select k objects from S , with repetition allowed, is

$$\binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1)!}.$$

Proof with binary strings.

Example. How many integer solutions are there to

$$x_1 + x_2 + x_3 + x_4 + x_5 = 10 \text{ with } x_i \geq 0 ?$$

Example. How many integer solutions are there to $x_1 + x_2 \leq 7$ with $x_1 \geq 0$ and $x_2 \geq 0$?

Example. How many ways are there to distribute 5 apples, 4 oranges and 3 pears to three people?

Example. Consider the following code segments.
What is the value of counter after the loops have executed ?

```
counter = 0;
for( i=1; i<=20; i++ )
    for( j=1; j<=20; j++ )
        for( k=1; k<=20; k++ )
            counter = counter + 1;
```

```
counter = 0;
for( i=1; i<=20; i++ )
    for( j=i; j<=20; j++ )
        for( k=j; k<=20; k++ )
            counter = counter + 1;
```

Example. A box contains 10 red balls, 10 green balls and 10 blue balls. Each set of balls is numbered 1 to 10. Suppose 7 balls are drawn at random from the box. In how many ways can there be 3 of one colour, 2 of a second colour and 2 of the 3rd colour.

Exercise. How many paths of length 2 edges are there in K_6 ?