MACM 202 Assignment 2, Fall 2005

This assignment is worth 10% of your grade. It is due Saturday October 8th at 5pm. A late penalty of 20% will apply for each weekday day late. Do each question in a separate Maple worksheet and hand in a printout of each worksheet.

Question 1 (15 marks) Do exercise 1.20 from the text. Do part (e) before part (c) so that you can see where the period doubling bifurcation point is. For part (e), use the bifurc.mws worksheet which has a Maple procedure for creating a bifurcation plot. For part (c), you will need to solve the system of equations $f(x) = x$ and $f'(x) = -1$ for a and x. Use `fsolve`. Also, graph the curves $f(x) = x$ and $f'(x) = -1$ using the `implicitplot` command from the plots package so that you can visually see where the solution is. See ?plots[implicitplot]).

Question 2 (15 marks) Find a value for the parameter a in the logistic map $f(x) = ax(1-x)$ where there is a stable 3-cycle. To help you locate the stable 3-cycle, generate a bifuration plot between $a = 3$ and $a = 4$. Now for a value of a where the 3-cycle is stable, plot x and $f(f(f(x)))$ then using `fsolve`, determine the values $x = y_1, y_2, y_3$ for the stable 3-cycle.

Now, using `fsolve`, determine the range of stability of the stable 3-cycle, i.e., find the range end points a_L and a_R such that for $a_L < a < a_R$ the 3-cycle is stable. To visualize the solution for a_R, use the implicitplot command in the plots package (see ?plots[implicitplot]) to graph the curves $g(x) = x$ and $g'(x) = -1$ where $g(x) = f(f(f(x)))$ on the same plot. Then use fsolve to compute a_L and a_R.

Question 3 (30 marks) Compute as many of the period doubling bifurcation points for the logistic map f$(x) = ax(1-x)$ that begin at $a = 3$ as you can. For full marks, compute up to the bifurcation point between the 512-cycle and the 1024-cycle. Now, estimate the value of Feigenbaum's constant from the data you obtain.
Note: this question is more difficult than the other questions.

Question 4 (20 marks) Design a boolean network with four nodes which counts in binary from 0 to 15 and then cycles. You may use more than four nodes if you wish but you must identify the four nodes which form the counter. Run your network using the `run` procedure in the networks.mws worksheet under the assignment lab directory and show the output.

Question 5 (20 marks) Consider a circular boolean network of $n \geq 3$ nodes numbered $1, 2, ..., n$, Suppose three of the nodes, say nodes $1, s, t$ where $1 < s < t \leq n$ apply the "Inverse" rule and the remaining $n - 3$ nodes apply the "Identity" rule. For "sufficiently many" values of (n, s, t) compute the length of the cycle resulting from the initial state $[0, 0, ..., 0]$. Use the `run2cycle` procedure in the networks.mws worksheet to do this. Study the data that you obtain and come up with a general rule, for the length of the cycle as a function of (n, s, t).