

# MACM 204 Makeup Assignment - Practice Final.

Fall 2017

Michael Monagan.

## You do not have to do this assignment.

It is meant to be a practice assignment, a warm up for the final exam.

However, if you got a bad mark on one of your assignments, or you missed an assignment, you may do this assignment and I'll replace your worst mark with your mark for this one.

Due Tuesday December 5th at 3pm.

There are 8 questions. There are 80 marks.

If you've forgotten how to use a command, just look at the examples in the help page for that command. If you get stuck, please feel free to come and ask me for help.

My office phone number is (778) 782 4279.

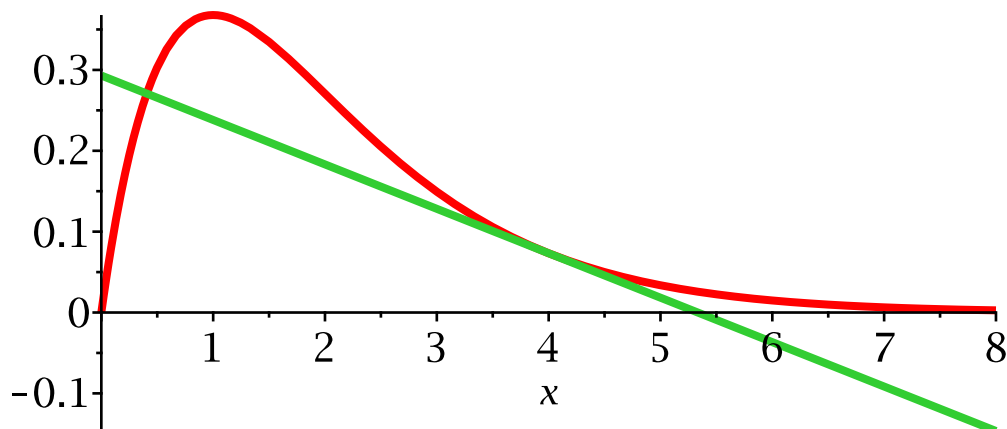
Please attempt each question in a separate worksheet.

Print your Maple worksheets (double sided if you wish) and hand them in to me.

**Reminder:** Please complete the course evaluation at <https://sfu.bluer.a.com/SFU>

## Question 1 (6 marks)

Part a) Shown below is a plot of  $f(x) = x \cdot \exp(-x)$  and the tangent line  $T(x)$  at  $x = 4$ . Recreate this plot.



## Question 2 (10 marks)

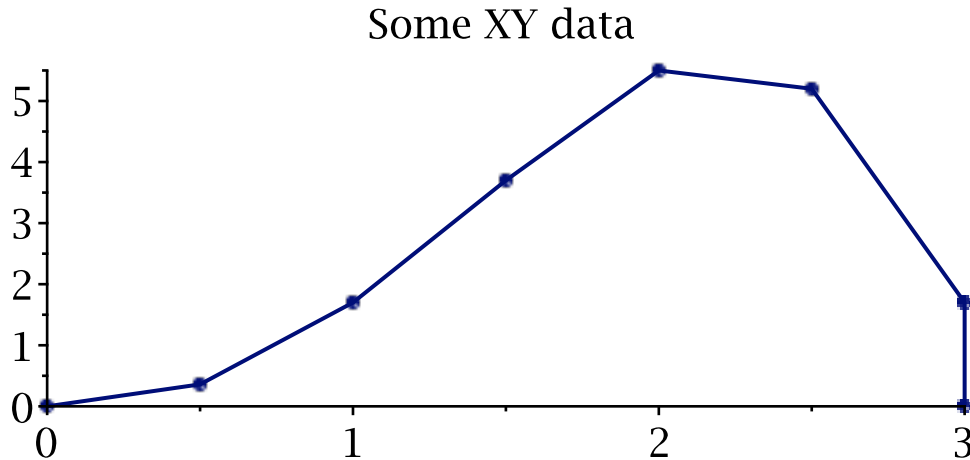
```
> Data := [[0.0,0.0], [0.50,0.36], [1.,1.7], [1.5,3.7],  
           [2.0,5.5], [2.5,5.2], [3.0,1.7], [3.0,0.0]];
```

```
Data := [[0., 0.], [0.50, 0.36], [1., 1.7], [1.5, 3.7], [2.0, 5.5], [2.5, 5.2], [3.0, 0.0]];
```

 (2.1)

1.7], [3.0, 0.]]

Below is a plot of the data points with lines between them.



Write a Maple procedure **TrapezoidalRule(Data)** that on input of a list of data points, each point of the form  $[x_i, y_i]$ , computes the area under the solid curve in the figure above. Test your Maple procedure on the data above. You should get 8.655.

### Question 3 (8 marks)

Consider the function  $f(x, y)$  below. Compute the formula for the tangent plane  $T(x, y)$  about  $x = 1, y = 1$ . Generate a plot of  $f(x, y)$  AND the tangent plane on the same plot using the **plot3d** command. Choose options for the tangent plane and  $f(x, y)$  so that each is distinct and easily visible.

```
> f := 1-x^2-y^2-3*x;
```

$$f := -x^2 - y^2 - 3x + 1$$

(3.1)

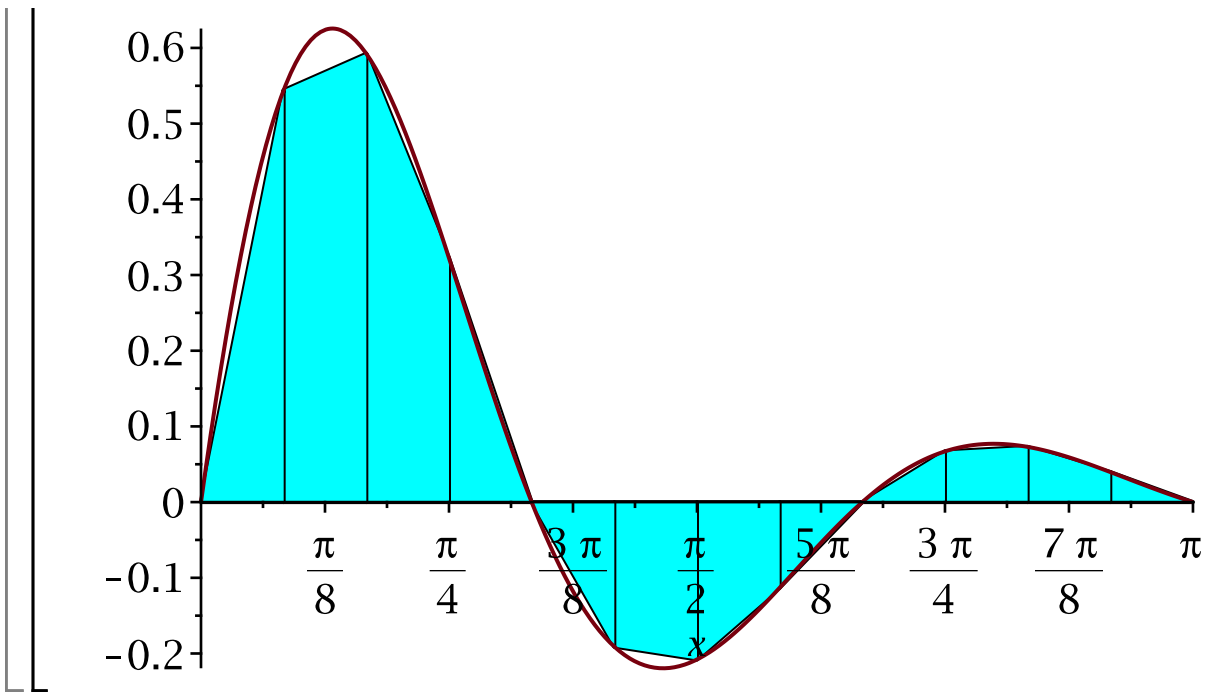
Now on a second plot, graph the surface  $f(x, y)$  and the curves  $f(1, y)$  and  $f(x, 1)$  on the same plot.

Use the **spacecurve** command from the **plots** package for the curves. Use  $0 \leq x \leq 2$  and  $0 \leq y \leq 2$  for the domain.

### Question 4 (12 marks)

Write a Maple procedure **TrapPlot** such that given a function  $f(x)$  on an interval  $x = a..b$  constructs  $n$  trapezoids of equal width  $h = (b - a)/n$  that approximate the area between  $f(x)$  and the  $x$  axis as shown in the figure below. Display also a plot of  $f(x)$  on the given interval. The inputs are **TrapPlot(f, x, a, b, n)**. Use the **plottools[polygon]** command to create a trapezoid. Test your procedure on this input.

```
> TrapPlot( exp(-x)*sin(3*x), x, 0, Pi, 12 );
```



### Question 5 (10 marks)

Consider the equilateral triangle  $A = (0, 0)$ ,  $B = \left(\frac{1}{2}, \sqrt{\frac{3.0}{4}}\right)$ ,  $C = (1, 0)$ .

The **Fractal game** is to do the following

Pick an initial point  $x_0$  anywhere in the triangle.

for  $k$  from 1 to  $n$  do

    Toss a die

    If you get 1 or 2 set  $x_k$  to be half way from  $x_{k-1}$  to  $A$ .

    If you get 3 or 4 set  $x_k$  to be half way from  $x_{k-1}$  to  $B$ .

    Otherwise set  $x_k$  to be half way from  $x_{k-1}$  to  $C$ .

This generates a sequence on  $n$  points  $x_1, x_2, \dots, x_n$  which must all be inside the triangle.

Program this in Maple and graph the points for  $n$  at least 5000. What picture do you get?

Note, to simulate a die use

```
> die := rand(1..6):
```

Then `die();` will generate a random integer on  $[1,6]$ . E.g.

```
> die();
```

4

(5.1)

```
> die();
```

3

(5.2)

```
> die();
```

(5.3)

### Question 6 (12 marks)

- a) calculate  $e^1$  and  $\ln(\pi)$  to 20 decimal places.
- b) evaluate and simplify the sum  $\sum_{i=0}^n i^2 \cdot 2^i$
- c) solve the linear system  $\{\alpha \cdot A - \beta \cdot B = 1, \beta \cdot A - \alpha \cdot B = 1\}$  for  $A$  and  $B$  using solve
- d) evaluate and simplify  $\int_{\alpha}^{\beta} (x - \alpha) \cdot (x - \beta) dx$
- e) solve the DE  $y'(t) = k \cdot y(t) - H$  using dsolve with initial value  $y(0) = 0$ .
- f) Consider the function  $f(x) = x^3 - 5 \cdot x^2 - 5$ .  
 Input  $f(x)$  as a Maple function (procedure).  
 Using the seq command construct the sequence  $f(1), f(2), f(3), \dots, f(9)$ .

### Question 7 (12 marks)

- Consider the function  $f(x) = \frac{\sin(x)}{x}$ .
- a) Graph  $y = f(x)$  and the line  $y = x$  on the same plot on the domain  $0 \leq x \leq 2$ .
- b) Solve  $f(x) = x$  to 5 decimal places.
- c) Calculate  $f'(1)$  to 5 decimal places.
- d) Evaluate  $\int_0^1 f(x) dx$  to 5 decimal places.
- e) Expand  $f(x)$  as a Taylor series to  $O(x^{10})$ .
- f) Calculate the  $\lim_{x \rightarrow 0} f(x)$  in Maple.

### Question 8 (10 marks)

Input the following function (correctly!)

$$f := x^4 - 2x^3y - 2xy^3 + y^4 + xy - x - y - 8 \quad (8.1)$$

It has one **real** critical point. To find it we can solve the system  $\{f_x(x, y) = 0, f_y(x, y) = 0\}$ . We will have to solve two simultaneous cubic equations

$$\begin{aligned} > \text{fx} := \text{diff}(f, x); \\ & \quad \quad \quad \text{fx} := 4x^3 - 6x^2y - 2y^3 + y - 1 \end{aligned} \quad (8.2)$$

$$\begin{aligned} > \text{fy} := \text{diff}(f, y); \\ & \quad \quad \quad \text{fy} := -2x^3 - 6xy^2 + 4y^3 + x - 1 \end{aligned} \quad (8.3)$$

(a) First, graph the curves  $f_x = 0$  and  $f_y = 0$  using the plots[implicitplot] command so you can see roughly where the real solution is.

(b) Next solve the equations using fsolve to find the critical point  $x = a, y = b$ .

(c) Calculate the partial derivatives  $f_{xx}(x, y)$  and  $f_{yy}(x, y)$  and  $f_{xy}(x, y)$  and determine what kind of critical point it is.

(d) Check that it is correct by graphing the surface  $f(x, y)$  and drawing a vertical line at  $x = a, y = b$  through the surface on a suitable domain so you can see the critical point clearly.