

How big can the coefficients of the factors of $x^n - 1$ in $\mathbb{Z}[x]$ be?

```
> f := x^30-1;
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$$f := x^{30} - 1$$

```
> F := [op(factor(f))];
```

$$F := [x-1, x^4+x^3+x^2+x+1, x^2+x+1, x^8-x^7+x^5-x^4+x^3-x+1, 1+x, x^4-x^3+x^2-x+1, x^2-x+1, x^8+x^7-x^5-x^4-x^3+x+1]$$

```
> S := combinat[subsets](F):
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```
> while not S[finished] do
  c := S[nextvalue](S);
  g := expand( mul(h,h=c) );
  if maxnorm(g)>7 then
    divide(f,g,'q');
    print(maxnorm(g),maxnorm(q));
    print(g,q);
  fi;
od;
```

$$12, 12$$

$$x^{15} + 4x^{14} + 8x^{13} + 10x^{12} + 8x^{11} + 2x^{10} - 6x^9 - 12x^8 - 12x^7 - 6x^6 + 2x^5 + 8x^4 + 10x^3 + 8x^2 + 4x + 1, x^{15} - 4x^{14} + 8x^{13} - 10x^{12} + 8x^{11} - 2x^{10} - 6x^9 + 12x^8 - 12x^7 + 6x^6 + 2x^5 - 8x^4 + 10x^3 - 8x^2 + 4x - 1$$

$$12, 12$$

$$x^{15} - 4x^{14} + 8x^{13} - 10x^{12} + 8x^{11} - 2x^{10} - 6x^9 + 12x^8 - 12x^7 + 6x^6 + 2x^5 - 8x^4 + 10x^3 - 8x^2 + 4x - 1, x^{15} + 4x^{14} + 8x^{13} + 10x^{12} + 8x^{11} + 2x^{10} - 6x^9 - 12x^8 - 12x^7 - 6x^6 + 2x^5 + 8x^4 + 10x^3 + 8x^2 + 4x + 1$$

$$8, 7$$

$$x^{16} - 3x^{15} + 4x^{14} - 2x^{13} - 2x^{12} + 6x^{11} - 8x^{10} + 6x^9 - 6x^7 + 8x^6 - 6x^5 + 2x^4 + 2x^3 - 4x^2 + 3x - 1, x^{14} + 3x^{13} + 5x^{12} + 5x^{11} + 3x^{10} - x^9 - 5x^8 - 7x^7 - 5x^6 - x^5 + 3x^4 + 5x^3 + 5x^2 + 3x + 1$$

$$8, 7$$

$$x^{16} + 3x^{15} + 4x^{14} + 2x^{13} - 2x^{12} - 6x^{11} - 8x^{10} - 6x^9 + 6x^7 + 8x^6 + 6x^5 + 2x^4 - 2x^3 - 4x^2 - 3x - 1, x^{14} - 3x^{13} + 5x^{12} - 5x^{11} + 3x^{10} + x^9 - 5x^8 + 7x^7 - 5x^6 + x^5 + 3x^4 - 5x^3 + 5x^2 - 3x + 1$$