2.3 Review of Vector Spaces

A vector space over F a field is a set V with two operations, addition $+$, scalar multiplication $\cdot$, satisfying the following axioms for all $x, y, z \in V$ and $a, b \in F$

1	$x+y \in V$	6	$a \cdot x \in V$
2	$x+y = y+x$	7	$1_F \cdot x = x$
3	$x+(y+z) = (x+y)+z$	8	$a\cdot(x+y) = a\cdot x + a\cdot y$
4	$\exists\, 0 \in V : x+0 = x$	9	$(a+b)\cdot x = a\cdot x + b\cdot x$
5	$\exists\, -x \in V : x+(-x) = 0$	10	$(a\cdot b)\cdot x = a\cdot(b\cdot x)$

The elements of F are called scalars and the elements of V are called vectors and multiplication of vectors is not required.

A set of vectors $B \subseteq V$ is a basis for V if B is linearly independent and spans V.

$\{v_1, v_2, \dots, v_n\}$ are linearly independent if for $a_1, a_2, \dots, a_n \in F$
$a_1v_1 + a_2v_2 + \dots + a_nv_n = 0 \Rightarrow a_1 = a_2 = \dots = a_n = 0.$

$\{v_1, v_2, \dots, v_n\}$ span V if for every $v \in V$, $\exists\, c_1, \dots, c_n \in F$ such that $c_1v_1 + c_2v_2 + \dots + c_nv_n = v.$

If B_1 and B_2 are bases for V then $|B_1| = |B_2|$
If B is a basis for V the dimension of V is $|B|$.