Example

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Example of Ben-Or Tiwari sparse polynomial interpolation.


```
> M1,M2,M3 := x^3*y^4, x*y^3*z, x^6*z^2;
```

$$M1, M2, M3 := x^3 y^4, x y^3 z, x^6 z^2$$

```
> a1,a2,a3 := 101,103,105;
```

$$a1, a2, a3 := 101, 103, 105$$

```
> f := a1*M1+a2*M2+a3*M3;
```

$$f := 105 x^6 z^2 + 101 x^3 y^4 + 103 x y^3 z$$

So f has t=3 terms. Assume we don't know t. Let's try T=4.

```
> T := 4;
```

$$T := 4$$

```
> for i from 0 to 2*T-1 do v[i] := eval( f, {x=2^i,y=3^i,z=5^i} )
  od;
```

$$v_0 := 309$$
$$v_1 := 261258$$
$$v_2 := 318719004$$
$$v_3 := 459589225992$$
$$v_4 := 706483640520816$$
$$v_5 := 1112692343818548768$$
$$v_6 := 1769125342359905801664$$
$$v_7 := 2823428649379900233478272$$

```
> H := Matrix([[v[0],v[1],v[2],v[3]],
               [v[1],v[2],v[3],v[4]],
               [v[2],v[3],v[4],v[5]],
               [v[3],v[4],v[5],v[6]]] );
```

$$H := \begin{bmatrix} 309 & 261258 & 318719004 & 459589225992 \\ 261258 & 318719004 & 459589225992 & 706483640520816 \\ 318719004 & 459589225992 & 706483640520816 & 1112692343818548768 \\ 459589225992 & 706483640520816 & 1112692343818548768 & 1769125342359905801664 \end{bmatrix}$$

```
> with(LinearAlgebra):
  Rank(H);
```

$$3$$

So we know t = 3.

```
> H := H[1..3,1..3];
```

$$H := \begin{bmatrix} 309 & 261258 & 318719004 \\ 261258 & 318719004 & 459589225992 \\ 318719004 & 459589225992 & 706483640520816 \end{bmatrix}$$

```
> S := -Vector([v[3],v[4],v[5]]);
```

$$S := \begin{bmatrix} -459589225992 \\ -706483640520816 \\ -1112692343818548768 \end{bmatrix}$$

```
> L := LinearSolve(H,S);
```

$$L := \begin{bmatrix} -279936000 \\ 1643760 \\ -2518 \end{bmatrix}$$

```
> Lambda := z^3+L[1]+L[2]*z+L[3]*z^2;
```

$$\Lambda := z^3 - 2518 z^2 + 1643760 z - 279936000$$

```
> factor(Lambda);
```

$$(z - 1600)\ (z - 270)\ (z - 648)$$

```
> R := roots(Lambda);
```

$$R := [[1600, 1], [270, 1], [648, 1]]$$

```
> m1,m2,m3 := seq( r[1], r in R );
```

$$m1, m2, m3 := 1600, 270, 648$$

```
> ifactor(m1),ifactor(m2),ifactor(m3);
```

$$(2)^6\ (5)^2, (2)\ (3)^3\ (5), (2)^3\ (3)^4$$

```
> M1,M2,M3 := x^6*z^2, x*y^3*z, x^3*y^4;
```

$$M1, M2, M3 := x^6 z^2, x y^3 z, x^3 y^4$$

The 3 by 3 Vandermonde system for the monomials is

```
> V := Matrix( [[1,1,1],[m1,m2,m3],[m1^2,m2^2,m3^2]] );
```

$$V := \begin{bmatrix} 1 & 1 & 1 \\ 1600 & 270 & 648 \\ 2560000 & 72900 & 419904 \end{bmatrix}$$

```
> b := <v[0],v[1],v[2]>;
```

$$b := \begin{bmatrix} 309 \\ 261258 \\ 318719004 \end{bmatrix}$$

```
> a := LinearSolve(V,b);
```

$$a := \begin{bmatrix} 105 \\ 103 \\ 101 \end{bmatrix}$$

```
> g := a[1]*M1+a[2]*M2+a[3]*M3;
```

$$g := 105 x^6 z^2 + 101 x^3 y^4 + 103 x y^3 z$$

```
> f;
```

$$105 x^6 z^2 + 101 x^3 y^4 + 103 x y^3 z$$