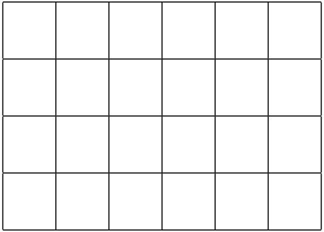
Lecture 2: Basic Counting Principles

Reading: Grimaldi Sections 1.1, 1.2

Lattice paths arise in theoretical physics.



How many lattice paths are there from $(0,0)$ to $(6,4)$ if we are restricted to North steps and East steps only?

Definition (Rule of Sum)

If there are m ways to perform task X and n ways to perform task Y, there are $m + n$ ways to perform **either** X or Y.

Definition (Rule of Product)

If there are m ways to perform task X and n ways to perform task Y, there are $m\,n$ ways to perform **both** X and Y.

Examples.

Exercise. If there are 10 people at a party and all hug each other, how many hugs are there?

Theorem (Strings)

If Σ is an alphabet with k letters, the number of strings of length n over Σ is k^n.

Proof.

Theorem (Permutations)

The number of permutations of a set of n distinct objects is $n!$.

Proof.

Definition (Permultations with Repetition)

Suppose there k_1 objects of type A, k_2 of type B, ..., and k_r of type R and let $n = k_1 + k_2 + \cdots + k_n$ be the total number of objects. The number of distinct permutations is denoted by $\binom{n}{k_1, k_2, \ldots, k_r}$.

Example. Consider the letters M, E, E, N, N. How many permutations are there?

Theorem (Permutations with Repetition)

$$\binom{n}{k_1, k_2, \ldots, k_r} = \frac{n!}{k_1!, k_2!, \ldots, k_r!}.$$

Proof.

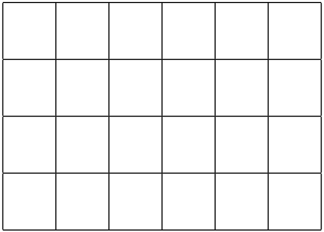
Exercise. How many binary strings of length 20 are there with exactly 13 1's?

Theorem (Subsets and Combinations)

If S is a set of size n, the number of subsets of size k $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Proof.

Lattice paths arise in theoretical physics.



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