

Lecture 14 Solving first order non-homogeneous RRs

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Grimaldi 10.3



Can you move the disks on pole 1 to pole 3 using pole 2 as needed?

Rule 1: move one disk at a time.

Rule 2: do not put a disk on top of a smaller disk.

Question: how many moves is necessary?

Solving first order non-homogeneous recurrences

Consider the non-homogenous recurrence relations

(1) $a_n + c_1 a_{n-1} = f(n)$ where $c_1 \neq 0$ and $f(n) \neq 0$

(2) $x_n + c_1 x_{n-1} + c_2 x_{n-2} = f(n)$ where $c_2 \neq 0$ and $f(n) \neq 0$

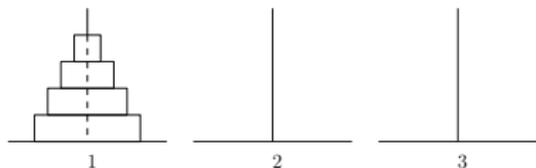
How do we solve them?

Case (1) $a_n + c_1 a_{n-1} = f(n)$ where $c_1 = -1$.

Example 1. Solve $a_n - a_{n-1} = 3n^2$ with $a_0 = 7$.

Example 2. Solve $a_n - 3a_{n-1} = 5 \cdot 3^n$ with $a_0 = 2$.

Example 3 – The Towers of Hanoi.



Move the disks from pole 1 to pole 3 using pole 2 as needed.

Move one disk at a time. Do not put a bigger disk on top of a smaller one.

(1)

(2)

(3)

Let m_n be the **number of moves**.

Determine and solve a recurrence relation for m_n .

Example 3 – The Towers of Hanoi (cont.)

Example 4 – Interest on a loan.

Pauline takes out a bank loan for $\$S$ dollars. She pays back $\$P$ every month and the bank charges her $r\%$ interest per month. Let a_n be the amount she owes after n months. Determine, and solve, a recurrence relation for a_n .

Example 4 – Interest on a loan (cont.)