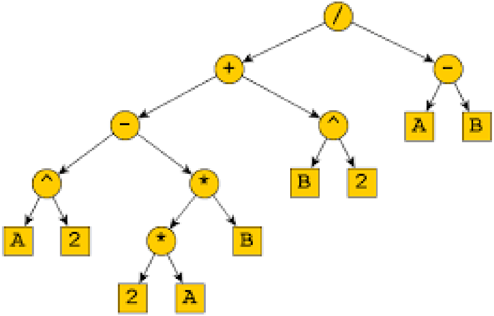
Lecture 30: Rooted Trees

Grimaldi 12.2



What formula does this tree encode?

Ordered rooted trees

For some applications it is essential to have not just a rooted tree, but also an ordering of the children for each internal vertex.

Example

- Thesis
  - Ch1
    - S1.1
    - S1.2
  - Ch2
    - S2.1
    - S2.2
      - S2.2.1
    - S2.3
  - Ch3

How do we walk through and process a rooted tree?

Definition (preorder, postorder tree traversals)

A **preorder traversal** of a tree T first visits the root vertex then visits, in preorder, the vertices of the subtrees $T_1, T_2, \ldots, T_k$ of T.
A **postorder traversal** of a tree T visits, in postorder, the vertices of the subtrees $T_1, T_2, \ldots, T_k$ of T then visits the root.

Example

Exercise Draw the expression tree for $(3 \times 5) + ((7 - 4) \times 2)$ and give the postorder traversal.

Preorder is also called Polish notation and postorder is also called reverse Polish notation. HP calculators used postorder and a stack to evaluate expressions.

Definition (spanning tree)

Let G be a connected multigraph. A subgraph T of G is a **spanning tree** if T spans G (so T contains all vertices in G) and T is a tree.

Example

Question How many spanning trees does C_n have?

Theorem (existence of spanning trees)

Every connected multigraph $G = (V, E)$ has a spanning tree.
Here are three algorithms to construct a spanning tree in G:

(1) *Start from G. If there is a cycle C in G delete an edge from C. Repeat this until G has no cycles. Output G.*

(2) *Create the graph $H = (V, \phi)$. For each edge e in G add e to H if it does not make H have a cycle. Output H.*

(3) *The depth-first-search algorithm.*

Proof (1)

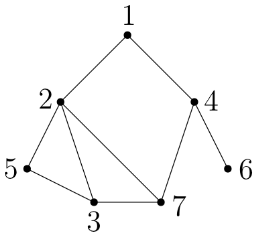
The Depth-First Search (DFS) algorithm

Input. A graph $G = (V, E)$.

Output. A set E_T of edges such that (V, E_T) is a spanning tree of G.

1. Let $v = 1$, $E_T = \phi$ and and mark vertex 1 as visited.
2. If all neighbors of v have been visited Then
   a) If $v = 1$ Then Return E_T.
   b) Else (backtrack step) Let $v = parent(v)$ and Goto step 2.
3. Else
   a) Let i be the smallest neighbor of v that has not been visited.
   b) Mark i as visited.
   c) Add the edge $\{v, i\}$ to E_T and Let $parent(i) = v$.
   d) Let $v = i$ and Goto step 2.

Example



Extra space.