Lecture 1: Fundamental Combinatorial Objects

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We will study four combinatorial objects

1. sets and subsets
2. strings and permutations
3. graphs
4. trees

Example Sets and Subsets

Strings

Definition (alphabet and string)

An **alphabet** Σ is a set of n elements called **letters**.
A **string** S of size n is an ordered sequence of n letters from Σ.

Examples $\Sigma = \{0, 1\}$ $\Sigma = \{A, C, G, T\}$

Exercise How many DNA sequences are there of length n ?

Example Find all strings of length 6 over $\{0, 1\}$ that don't have 10 as a substring.

Permutations

Definition (permutation)

A **permutation** P over an alphabet Σ is a string over Σ where every letter occurs exactly once.

Example $\Sigma = \{1, 2, 3\}$ find all permutations.

Theorem

The number of permutations of a set of n objects is $n!$.

Graphs

Definition (graph)

A (simple) **graph** G is a pair (V, E) where V is a set of **vertices** and E is a set of unordered pairs of vertices called **edges**. If $e = \{i, j\} \in E$ we say vertices i and j are **adjacent**. The **degree** of a vertex is the number of adjacent vertices.

Example $V = \{1, 2, 3, 4, 5, 6\}$,
$E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{1, 5\}, \{2, 5\}, \{4, 6\}\}$

Question. How many edges can a graph with n vertices have?

Definition (complete graph)

A graph $G = (V, E)$ is **complete** if $|V| \geq \mathbf{1}$ and for all $i, j \in V$ the edge $\{i, j\} \in E$. The complete graph with n vertices is denoted K_n.

Definition (path graph)

A graph $G = (V, E)$ is a **path** if $|V| \geq \mathbf{1}$ and V may be ordered $v_1, v_2, \ldots, v_n$ so that $E = \{\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}\}$. The path graph with n vertices is denoted P_n.

Definition (cycle graph)

A graph $G = (V, E)$ is a **cycle** if $|V| \geq \mathbf{3}$ and V may be ordered $E = \{\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}, \{v_n, v_1\}\}$. The cycle graph with n vertices is denoted C_n.

Examples

Definition (connected graph)

A graph $G = (V, E)$ is **connected** if there is a path in G from vertex $i \in V$ to vertex j for all $i \neq j$.

Definition (tree)

A graph $G = (V, E)$ is a **tree** if it is connected and has no cycles.

Example. All (unlabelled) trees with 4 vertices.

Exercise. Draw all (unlabelled) trees with 5 vertices.

Exercise. If G is a tree with $n > 0$ vertices, how many edges must G have?