Lec 0 Sums and Products Review
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Calculus : Derivatives & Integrals & DEs

$\frac{d}{dx}\sin x$ $\quad \int e^x dx = e^x + C \quad y'' = k \cdot y.$

Discrete Math : Sums & Products & RRs & GFs

$$\sum_{i=1}^{n} i = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$$\prod_{i=1}^{n} i = 1\cdot 2\cdot 3 \cdot \cdots \cdot n = n!$$

Definition. $$\sum_{i=m}^{n \leftarrow \text{upper limit}} f(i) = \overset{i=m}{f(m)} + \overset{i=m+1}{f(m+1)} + \cdots + \overset{i=n}{f(n)}$$

summation index $\rightarrow i=m$

$\uparrow$ lower limit

$$\prod_{i=m}^{n} f(i) = f(m)\cdot f(m+1) \cdot \cdots \cdot f(n).$$

E.g. $$\prod_{i=1}^{3} x-i = (x-1)(x-2)(x-3).$$

Properties $$\sum_{i=1}^{n} c\cdot f(i) = c\cdot f(1) + c\cdot f(2) + \cdots + c\cdot f(n)$$

$$= c\,[\underbrace{f(1)+f(2)+\cdots+f(n)}]$$

$$= c \sum_{i=1}^{n} f(i)$$

$$\prod_{i=1}^{n} c\cdot f(i) = c\cdot f(1)\cdot c\cdot f(2) \cdot \cdots \cdot c\cdot f(n)$$

$$= c^n \prod_{i=1}^{n} f(i).$$

$$2\sum_{i=1}^{n} i = \begin{array}{ccccccccccc} 1 & + & 2 & + & 3 & + \cdots + & (n-1) & + & n \\ n & + & (n-1) & + & (n-2) & + \cdots + & 2 & + & 1 \end{array}$$

$$n+1 + n+1 + n+1 + \cdots + n+1 + n+1$$

$$= n(n+1).$$

$\Rightarrow$ $1.\ \sum^{n} i = \frac{n(n+1)}{2}$ Memorize.

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Define $n! = n\cdot(n-1)\cdot(n-2)\cdot\dots\cdot 3\cdot 2\cdot 1$

$0! = 1$

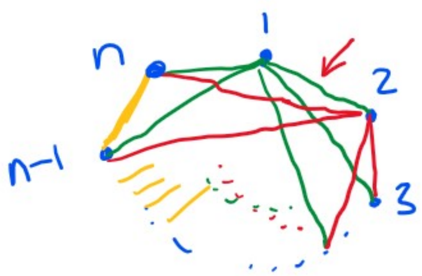
$3! = 3\cdot 2\cdot 1 = 6$

$4! = 4\cdot\underbrace{3\cdot 2\cdot 1}_{3!} = 24$

$n! = n\cdot(n-1)!$

Example $\prod_{i=1}^{n} 2i \overset{\text{Prop.}}{=} 2^n \prod_{i=1}^{n} i = 2^n\cdot n!$

Suppose n people are at a party and they hug each other.
How many hugs are there?



person 1 person 2 person 3 ...... n-1 n

$(n-1) + (n-2) + (n-3) + \dots + 1 + 0$

$= \sum_{i=1}^{n-1} i = \sum_{i=1}^{n} i - n$

$= \frac{n(n+1)}{2} - n$

$= n\left[\frac{n+1}{2} - 1\right] = n\left(\frac{n-1}{2}\right)$

$\sum_{i=1}^{n} i = \underbrace{1+2+\dots+n-1}_{\sum_{i=1}^{n-1} i} + n$

Combinatorial solution:

How many pairs of people are there at the party of n people?

There are $\binom{n}{2}$ or $nC2$ pairs. $\Rightarrow$ $\binom{n}{2}$ hugs.

$$\binom{n}{2} = \frac{n! = n(n-1)! = n(n-1)\cdot\cancel{(n-2)!}}{\underset{\overset{\shortparallel}{2}}{2!}\,\cancel{(n-2)!}} = \frac{n(n-1)}{2}.$$