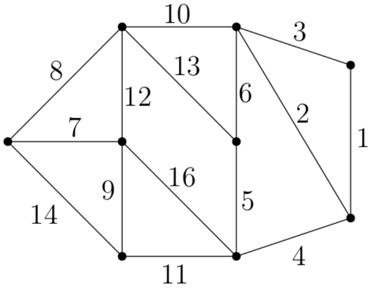
Lecture 32: Weighted Graphs and Minimum Spanning Trees

Grimaldi 13.2



Definition (Weighted Graph)

A **weighted graph** $G = (V, E)$ is a multigraph together with a function $w : E \to \mathbb{R}^+$ is called an **edge-weighting**.

Examples

Definition (Minimum Spanning Tree)

Let $G = (V, E)$ be a connected multigraph with edge-weighting w.
For any subgraph $H = (V', E')$ of G, the **weight** of H is

$$w(H) = \sum_{e \in E'} w(e).$$

A **minimum spanning tree** is a spanning tree of G of minimum weight.

Example.

Lemma (property of minimum spanning trees)

Let $G = (V, E)$ be a weighted connected graph. Let V_1 and V_2 be a partition of V. Amongst the edges in G with one vertex in V_1 and the other in V_2 let e one of minimum weight. There is a minimum spanning tree in G with e as one of it's edges.

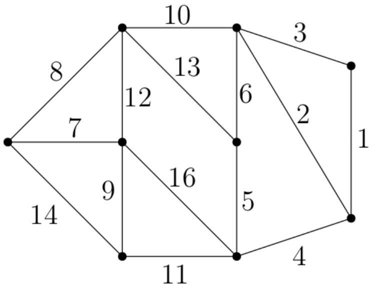
Proof.

Kruskal's algorithm to compute a minimum spanning tree

Input: a connected multigraph $G = (V, E)$ with an edge-weighting w.
Output: a mimimal spanning tree of G.

1. Set $E' = \phi$.
2. Sort the edges in E from least weight to highest weight.
3. While (V, E') is not connected do
   Let e be the next heaviest edge in E.
   If $(V, E' \cup \{e\})$ does not have a cycle set $E' = E' \cup \{e\}$.
4. Return the tree (V, E').

Example



Additional Space.

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