Lecture 6: Graph Isomorphism

Grimaldi 11.2
Which of the following graphs are the "same" ?
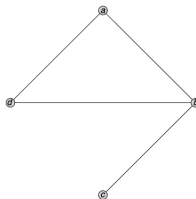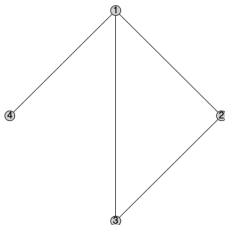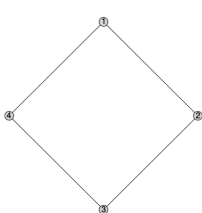
Definition (isomorphic graphs)

Let $G = (V_1, E_1)$ and $H = (V_2, E_2)$ be two graphs. Then G is **isomorphic** to H (has the same structure as) if there is a bijection $f : V_1 \to V_2$ such that

$$\{u, v\} \in E_1 \iff \{f(u), f(v)\} \in E_2.$$

The function f is called an **isomorphism**.

Example.

Example. Draw all non-isomorphic graphs with $|V| = 3$ and $|V| = 4$.

Exercise. Draw all non-isomorphic graphs with 5 vertices and 4 edges.

How can we test if two graphs G and H are isomorphic?

An "efficient" graph isomorphism algorithm is not known.

Example. For $n \geq t$, how many subgraphs of K_n are isomorphic to K_t ?

Example. Let K_4^- be K_4 less one edge.
How many subgraphs of K_n are isomorphic to K_4^-?

Example. How many subgraphs of $K_{n,m}$ are isomorphic to $K_{3,4}$?