Lec30 Rooted Trees and Spanning Trees Michael Monagan

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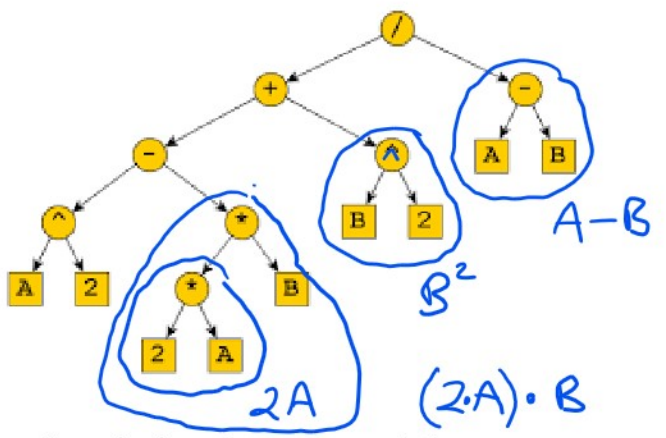
Lecture 30: Rooted Trees

Grimaldi 12.2



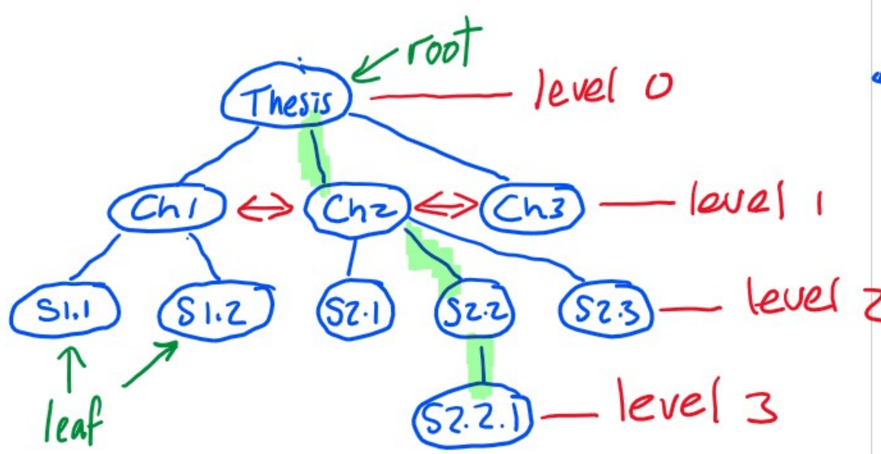
What formula does this tree encode?

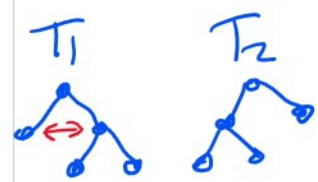
Ordered rooted trees

For some applications it is essential to have not just a rooted tree, but also an ordering of the children for each internal vertex.

Example

Thesis
- Ch1
  - S1.1
  - S1.2
- Ch2
  - S2.1
  - S2.2
    - S2.2.1
  - S2.3
- Ch3





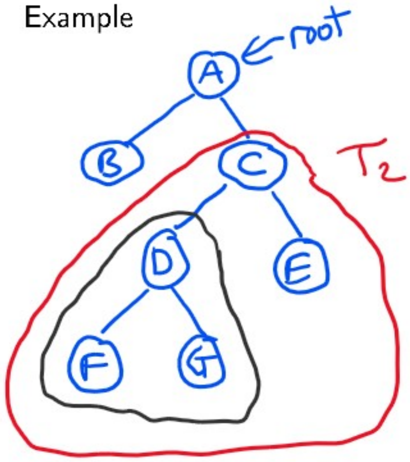
Height = 3 = length of the longest path from a leaf to the root

How do we walk through and process a rooted tree?

Definition (preorder, postorder tree traversals)

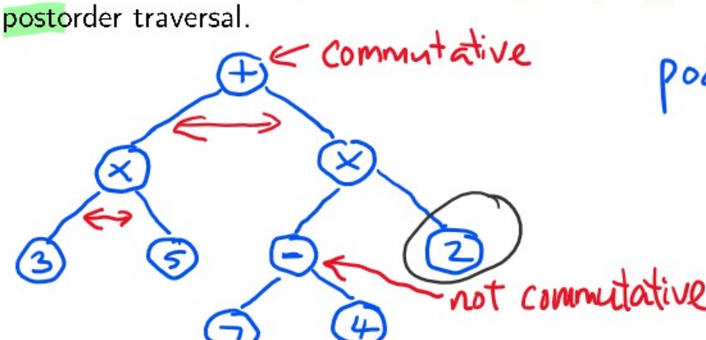
A **preorder traversal** of a tree T first visits the root vertex then visits, in preorder, the vertices of the subtrees $T_1, T_2, \ldots, T_k$ of T.
A **postorder traversal** of a tree T visits, in postorder, the vertices of the subtrees $T_1, T_2, \ldots, T_k$ of T then visits the root.

Example



preorder : A B C D F G E
postorder : B F G D E C A

Exercise. Draw the expression tree for $(3 \times 5) + ((7 - 4) \times 2)$ and give the postorder traversal.

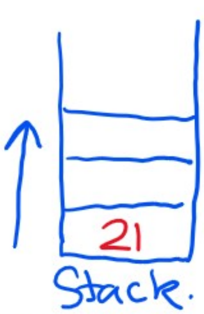


postorder : 3 5 × 7 4 − 2 × +

Preorder is also called Polish notation and postorder is also called reverse Polish notation. HP calculators used postorder and a stack to evaluate expressions.

3 5 × 7 4 − 2 × +

Rules. Number? Put it on top of the stack.
Operator? Take the top two items off the stack, operate, put the result on top of the stack.

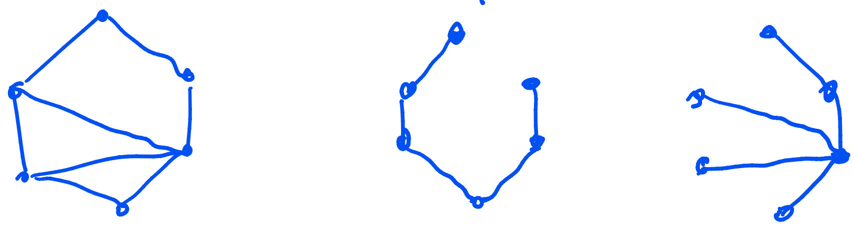


15+6=21

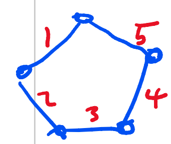
Definition (spanning tree)

Let G be a connected multigraph. A subgraph T of G is a **spanning tree** if T spans G (so T contains all vertices in G) and T is a tree.

Example G connected. T_1 T_2



C_5



How many spanning trees does C_n have? $\underline{\underline{n}}$

Theorem (existence of spanning trees)

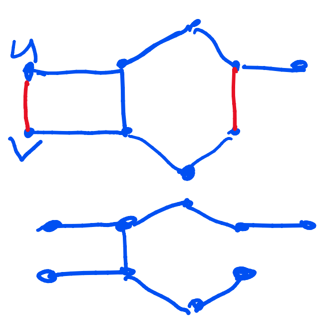
Every connected multigraph $G = (V, E)$ has a spanning tree.
Here are two algorithms to select a spanning tree in G:

(1) *Start from G. If there is a cycle C in G delete an edge from C. Repeat this until G has no cycles. Output G.*

(2) *Create the graph $H = (V, \phi)$. For each edge e in G add e to H if it does not make H have a cycle. Output H.*

Proof (sketch).

G.



Proof (2) is a exercise.

Proof (1)

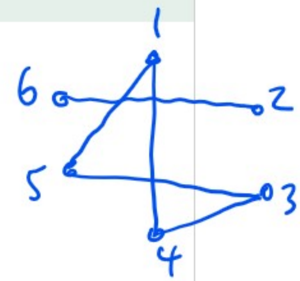
Deleting an edge on a cycle of E preserves connectivity.

End up with a connected graph with no cycles.

A tree with all vertices. This is a spanning tree.

Graph Algorithms

Consider the graph $G = (V, E)$ where $V = \{1, 2, 3, 4, 5, 6\}$ and $E = \{\{1, 5\}, \{5, 3\}, \{4, 1\}, \{4, 3\}, \{2, 6\}\}$.



How can a computer test if G is connected? planar?
If G is connected, how can it find a spanning tree in G?
If G is planar, how can it find a planar embedding of G?

Most algorithms need to visit the **neighbors** of a vertex.
Question: What is a good way to store the edges?

Definition (list of neighbors)

The **list of neighbors** representation for E is an array A of size $n = |V|$ where A_i is the set of neighbors of vertex i (the vertices adjacent to i).

Example

A

1	2	3	4	5	6
{4,5}	{6}	{4,5}	{1,3}	{1,3}	{2}

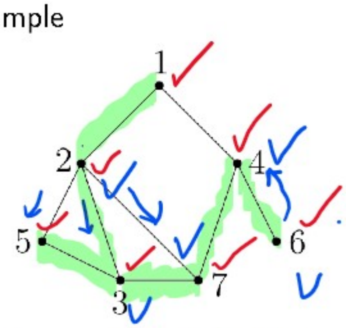
Question: How do algorithms walk through the graph?

The Depth-First Search (DFS) algorithm

`Input.` A graph $G = (V, E)$.
`Output.` A set E_T of edges such that (V, E_T) is a spanning tree of G.

1. `Let` $v = 1$, $E_T = \phi$ and and mark vertex 1 as visited.
2. `If` all neighbors of v have been visited `Then`
   a) `If` $v = 1$ `Then Return` (V, E_T).
   b) `Else` (backtrack step) `Let` $v = parent(v)$ and `Goto` step 2.
3. `Else`
   a) `Let` i be the smallest neighbor of v that has not been visited.
   b) Mark i as visited.
   c) Add the edge $\{v, i\}$ to E_T and `Let` $parent(i) = v$.
   d) `Let` $v = i$ and `Goto` step 2.

Example



$E_T = \{1,2\}, \{2,3\}, \{3,5\}, \{3,7\}, \{4,7\}, \{4,6\}$

parent

1	2	3	4	5	6	7
	1	2	7	3	4	3