Lecture 22: The Summation Operator

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Problem: $\sum_{i=1}^{n} i^2 = ?$ $\sum_{i=0}^{n} i^3 = ?$ $\sum_{i=1}^{n} i^4 = ?$ etc.

Let $A(x) = a_0 + a_1 x + a_2 x^2 + \dots$. What is

$$A(x)\frac{1}{1-x} =$$

Example 1. For $A(x) = x + x^2$ find $\dfrac{A(x)}{(1-x)}, \dfrac{A(x)}{(1-x)^2}$ and $\dfrac{A(x)}{(1-x)^3}$.

Example 2. We will find a formula for

$$\sum_{k=0}^{n} k^2 = 0^2 + 1^2 + 2^2 + 3^2 + \cdots + n^2.$$

Example 2 (cont.)

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Exercise 1. Use the summation operator to show $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

Proofs by induction.

Example 1. Show that $\sum_{k=1}^{n} 2k - 1 = n^2$ for $n \geq 1$ by induction on n.

Example 1 continued.

Exercise 2. Show that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ for $n \geq 1$ by induction on n.

Example 2. Show that every integer $n \geq 2$ can be factored into a product of primes.

Example 2 continued.