Lecture 24: Eulerian Trails and Circuits

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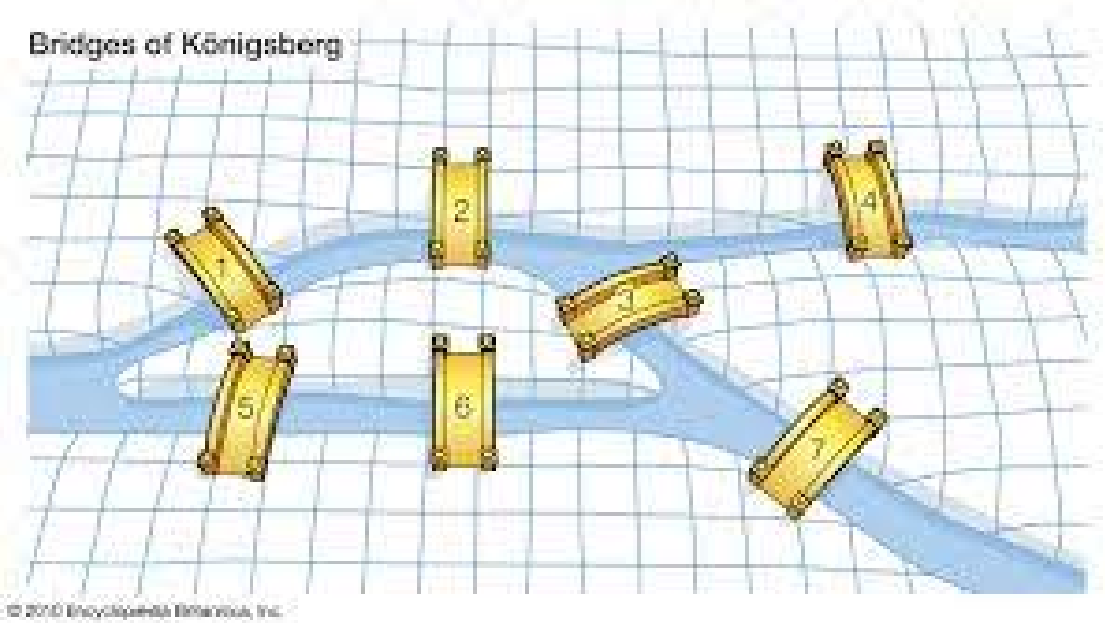

Question: Is it possible walk around the city crossing each bridge exactly once, and end up where you started?

Definition (Eulerian circuit)

An **Euler ciruit** of a multi-graph $G = (V, E)$ is a circuit

$$W = v_1, e_1, v_2, e_2, \ldots, e_n, v_1$$

such that every edge in E appears once in W.

Examples.

Lemma

Let $G = (V, E)$ be a multigraph with $|E| \geq 1$.
If $\deg(v) \geq 2$ for all $v \in V$, then G contains a cycle of length ≥ 1.

Proof.

Proof (cont).

Theorem (Euler)

A connected multigraph $G = (V, E)$ which is not the singleton vertex, has an Euler circuit if and only if every vertex in V has even degree.

Proof.

Proof (cont.)

Proof (cont.)

The proof gives a recursive algorithm for finding an Euler circuit!

Definition

An **Euler trail** of a multi-graph $G = (V, E)$ is a trail

$$T = v_0, e_1, v_1, e_2, \ldots, e_n, v_n$$

such that every edge in E appears once in T.

Example

Corollary (of Euler's theorem)

A connected multigraph $G = (V, E)$ has an Euler trail if and only if there are exactly two vertices in G of odd degree.

Proof. Exercise.