Lecture 11 Recurrence Relations

Grimaldi Chapter 10 Recurrence Relations

The Fibonacci sequence $1, 1, 2, 3, 5, 8, \ldots$ is generated by the **recurrence**

$$f_{n+1} = f_n + f_{n-1} \ \text{ for } n \geq 2$$

and **initial values**

$$f_1 = 1,\ f_2 = 1.$$

Example 2. Let b_n be the number of binary strings of length n bits.

A new way: To construct a binary string of length n first construct one of length $n-1$ bits.

Example 3. Let k_n be the number of edges in K_n the complete graph on n vertices.

Definition

A **linear** recurrence relation (RR) of **order** k with **constant coefficients** for a sequence $a_1, a_2, a_3, \ldots$ is an equation of the form

$$c_0 a_n + c_1 a_{n-1} + \cdots + c_k a_{n-k} = f(n) \text{ for } n \geq k$$

where $c_0, c_1, \ldots, c_k$ are constants and $c_0 \neq 0, c_k \neq 0$. If $f(n) = 0$ the RR is said to be **homegeneous**, otherwise it is **non-homogeneous**.

Examples

Note: We can **shift** a RR up or down without changing the solutions. For example

$$\begin{aligned} a_{n+1} &= 2a_n + n \\ a_n &= 2a_{n-1} + n - 1 \\ a_{n+2} &= 2a_{n+1} + n + 1 \end{aligned}$$

Initial Values:

Example 4. Let S_n be the set of all binary strings of length n with the property that every 1 is followed by a 0 (so 1 cannot be the last bit).

(1) List S_1, S_2, S_3.
(2) Let $c_n = |S_n|$. Give a recurrence relation for c_n.

Example 5. Let D_n be the set of all strings of length n over the alphabet $\Sigma = \{A, B, C, D\}$ such that every A is followed by a C and every B is followed by DD.

(1) List D_0, D_1, D_2.
(2) Let $d_n = |D_n|$. Give a recurrence relation for d_n.

Additional space