

## Lecture 4: Combinations with repetition: Grimaldi 1.4

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How many combinations of size 3 are there from  $S = \{a, b, c\}$  if repetitions are allowed?

## Theorem ( combinations with repetitions )

*Let  $S$  be a set with  $n$  elements. The number of ways to select  $k$  objects from  $S$ , with repetition allowed, is*

$$\binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1)!}.$$

**Proof with binary strings.**

Example. How many integer solutions are there to

$$x_1 + x_2 + x_3 + x_4 + x_5 = 10 \text{ with } x_i \geq 0 ?$$

Example. How many integer solutions are there to  $x_1 + x_2 \leq 7$  with  $x_1 \geq 0$  and  $x_2 \geq 0$ ?

Example. How many ways are there to distribute 5 apples, 4 oranges and 3 pears to three people?

Example. Consider the following code segments.  
What is the value of counter after the loops have executed ?

```
counter = 0;
for( i=1; i<=20; i++ )
    for( j=1; j<=20; j++ )
        for( k=1; k<=20; k++ )
            counter = counter + 1;
```

```
counter = 0;
for( i=1; i<=20; i++ )
    for( j=i; j<=20; j++ )
        for( k=j; k<=20; k++ )
            counter = counter + 1;
```

Example. A box contains 10 red balls, 10 green balls and 10 blue balls. Each set of balls is numbered 1 to 10. Suppose 7 balls are drawn at random from the box. In how many ways can there be 3 of one colour, 2 of a second colour and 2 of the 3rd colour.

Exercise. How many paths of length 2 edges are there in  $K_6$ ?