

Computing Tutte polynomials

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Abstract:

The Tutte polynomial from graph theory is the most general graph invariant that can be computed by edge deletion and contraction. Because it includes the Chromatic polynomial as a special case, it is NP-hard to compute, and a straight-forward implementation of the edge deletion and contraction algorithm, like the one in Mathematica, takes $O(1.6^{(n+m)})$ time where $n = |V|$ and $m = |E|$. In recent work, Haggard, Pearce and Royle studied edge selection heuristics and used the graph isomorphism test in Brendan McKay's "nauty" package to speed up the edge deletion contraction algorithm. They succeeded in computing the Tutte polynomial of the truncated icosahedron, a cubic graph with 32 vertices and 48 edges, in one week on 150 cores. See

<http://www.homepages.ecs.vuw.ac.nz/~djp/tutte>

In this talk we will present a new edge selection heuristic which we have implemented in vanilla Maple. It dramatically improves the computation time and surprisingly, computes the Tutte polynomial of some generalized Petersen graphs in polynomial time. The edge selection problem is reminiscent of the problem of choosing a good monomial ordering when computing Groebner bases. We will present also the connection of the Tutte polynomial with the reliability polynomial from network theory which is what motivated us to consider computing Tutte polynomials and we comment about what facilities in Maple aided and hindered the implementation.

No prior knowledge of Tutte polynomials is assumed.