

## Assignment 3 Question 2

```
> `mod` := mods;
                                mod:= mods
> a1 := 58*x^4-415*x^3-111*x+213;
                                a1:= 58x4 - 415x3 - 111x + 213
> b1 := 69*x^3-112*x^2+413*x+113;
                                b1:= 69x3 - 112x2 + 413x + 113
> gcd(a1,b1);
                                1
```

```
> p1 := 23;
                                p1:= 23
```

```
> Gcd(a1,b1) mod p1;
                                1
```

Since  $p_1=23$  is not a bad prime (it does not divide  $\text{lc}(a_1) = 58$ ) then this proves that  $a_1$  and  $b_1$  are relatively prime over the integers.

```
> a2 := x^5-111*x^4+112*x^3+8*x^2-888*x+896;
                                a2:= x5 - 111x4 + 112x3 + 8x2 - 888x + 896
> b2 := x^5-114*x^4+448*x^3-672*x^2+669*x-336;
                                b2:= x5 - 114x4 + 448x3 - 672x2 + 669x - 336
```

For this prime we know the gcd is monic and there are no bad primes since  $\text{lc}(a_2) = 1$

```
> p1 := 23;
                                p1:= 23
> g1 := Gcd(a2,b2) mod p1;
                                g1:= x2 + 4x - 3
```

```
> p2 := 29;
                                p2:= 29
> g2 := Gcd(a2,b2) mod p2;
                                g2:= x3 + 7x2 + 6x - 8
```

Thus  $p_2 = 29$  must be unlucky.

```
> p3 := 31;
                                p3:= 31
> g3 := Gcd(a2,b2) mod p3;
                                g3:= x3 - 8x2 - 6x + 4
```

That one must be unlucky too!

```
> p4 := 37;
                                p4:= 37
```

```
> g4 := Gcd(a2,b2) mod p4;
```



```

> g12 := chrem([g1,g2],[p1,p2]);
                 $g12 := 6x^3 + 48x - 34$ 
=
> p3 := 31;
                 $p3 := 31$ 
=
> g3 := alpha * Gcd(a3,b3) mod p3;
                 $g3 := 6x^3 - 14x - 3$ 
=
This time I will avoid combining g1 and g2.
=
> g123 := chrem([g12,g3],[p1*p2,p3]);
                 $g123 := 6x^3 + 48x - 34$ 
=
> g := primpart(g123);
                 $g := 3x^3 + 24x - 17$ 
=
> divide(a3,g);
                 $true$ 
=
> divide(b3,g);
                 $true$ 

```

[Thus the  $\gcd(a3,b3) = 3x^3 + 24x - 17$ .