Assignment 2, MACM 204, Fall 2012

Due Wednesday October 10th at 1:30pm at the beginning of the first lab.
Late penalty: -20% for up to 24 hours late. 0 after that.

Michael Monagan.

Please attempt each question in a seperate worksheet (so that you don't destroy your previous work). Print your Maple worksheets (you may print double sided if you wish) and hand them in to me.

There are 8 questions.

The purpose of this assignment is to get you to use the 3 - dimensional graphics commands, practice your programming skills, write some recursive programs, compute and visualize partial derivatives, and use Taylor series.

Question 1

Consider the function $g(a, x) = a^2 \cdot x \cdot (1 - x) \cdot (1 - a \cdot x + a \cdot x^2)$. We want to study the solutions of $g(a, x) = x$ for $0 \le a \le 4$ and $0 \le x \le 1$.
One way is to create an implicit plot of $g(a, x) = x$. Do this using the **implicitplot** command in the **plots** package.

Another way is to graph the function $g(a, x) - x$ in 3 dimensions for $0 \le a \le 4$ and $0 \le x \le 1$ and see where the *z* co-ordinate is 0. To do this visually we can graph the 0 function. Do this using the **plot3d** command (graph $g(a, x) - x$ and 0 on the same plot) . Rotate the plot so that it matches the implicit plot.

Solution 1

Question 2

Consider the function $f(x, y) = 2 \cdot x^2 + 3 \cdot y^2 - x \cdot y - 4$.
We want to visualize the partial derivatives at the point $x = 1, y = 1$.
First use Maple to compute the partial derivatives $\frac{\partial}{\partial x} f(1, 1)$ and $\frac{\partial}{\partial y} f(1, 1)$. You should get 3 and 1 respectively so both slopes are positive.
Now generate a 3 dimensional plot of $f(x, y)$ (using the **plot3d** command) and the curves $f(x, 1)$ and $f(1, y)$ (using the **spacecurve** command) and display all three plots on the same graph (using the **display** command).

Solution 2

Question 3

Consider the function $f(x, y) = 2 \cdot x^4 + 3 \cdot y^4 - x \cdot y - 4$.
We would like to visualize the tangent plane at $x = 1, y = 1$.

Use Maple to construct the linear Taylor polynomial T for $f(x, y)$ about $x = 1$, $y = 1$. Graph $f(x, y)$ and $T(x, y)$ on the same plot using the plot3d command.

Now use Maple to construct the quadratic Taylor polynomial Q for $f(x, y)$ about $x = 1$, $y = 1$. This will be T + terms in $(x-1)^2$, $(y-1)^2$ and $(x-1)\cdot(y-1)$. We could graph $f(x, y)$ and $Q(x, y)$ on the same graph to check that Q is an approximation of $f(x, y)$ but I found it to be a bit messy. I suggest you graph the error $f(x, y) - Q(x, y)$ instead.

Solution 3

Question 4

Given a list **L** of values and a value *x*, write a Maple procedure

    position := proc(x::anything,L::list) ... end;

so that position(x,L) returns the position (the index) of the first occurence of *x* in **L**. If *x* is not in **L**, return 0. For example for

```
> L := [1,5,4,6,2,4];
```

$$[1, 5, 4, 6, 2, 4] \tag{7.1}$$

```
> position(4,L);
```

should return 3.

Solution 4

Question 5

Given a list *x* = $[x_1, x_2, x_3, ..., x_n]$ write a Maple procedure Sn that outputs the sum of the first $n \geq 0$ values of *x*.

Your procedure should work by recursively adding the first *n*/2 values then recursively adding the second *n*/2 values Test your procedure on x = [1,2,3,4,5,...,10].

Solution 5

Question 6

The Taylor series for e^x is given by $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + ...$

For $|x| \leq 0.1$ the term in x^5 satisfies $\left|\frac{x^5}{120}\right| < 10^{-7}$. Since the remaining terms are negligible, if we use the Taylor polynomial $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$ to approximate e^x, then for $|x| \leq 0.1$, we will get seven decimal places of accuracy. But what if $|x| > 0.1$?

For $x > 0.1$ we can use the identity $e^{2 \cdot x} = (e^x)^2$. We first compute $y := e^{\frac{x}{2}}$ then compute y^2 to get e^x. Now if $\frac{x}{2}$ is still bigger than 0.1 we can divide by 2 again.

For $x < 0.1$ we can use the identity $e^{-x} = \frac{1}{e^x}$. Putting all of this together we can compute e^x for all x.

Write a Maple procedure EXP := proc(x::numeric) ... end that computes e^x accurately to 10 decimal places for $-\infty < x < \infty$. Can you do this with one division?

Solution 6

Question 7

The binomial coefficient $\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$. To compute it we could program $n!$ then code (in Maple)

```
BIN := proc(n,k) iquo(n!,k!*(n-k)!) end;
```

One problem with this is that it computes $n!$ which is a very big integer. This will work fine in Maple because Maple supports long integers but if we are programming in C or C++ or Java, even 21! will overflow on a 64 bit computer. But the biggest binomial coefficient for n=21 is 352716. The binomial coefficients are much smaller than n! In fact they add up to 2^n. A fact that Maple knows

```
> sum(binomial(n, k), k = 0 .. n);
```

$$2^n \qquad \textbf{(13.1)}$$

We will develop a better method. Find a formula for $\binom{n}{k}$ of the form

$$\binom{n}{k} = something \cdot \binom{n}{k-1}$$

where the something does not involve factorials. Now program a recursive Maple procedure BIN that uses this formula. Test your procedure on BIN(6,3), BIN(6,6), and BIN(20,10). Observe that to compute $\binom{n}{k}$ your procedure will compute all of

$\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{k-1}$ as well.

Insert a print command to print all the binomial coefficients that you compute.

Solution 7

Question 8

Consider a random walk in the XY plane where at each time step you walk one step (one unit) either to the left, right, up or down, at random. Starting from the origin, generate plots for at least two random walks with at least n=1000 random steps.

So first create a list of n values P := $[[0, 0], [x_1, y_1], [x_2, y_2], ..., [x_n, y_n]]$.
Then you can simply graph them using the plot(P, style=line); command.
To get random numbers from 1,2,3,4 use the following

```
> R := rand(1..4):
```

Now when you call R() you will get one of 1,2,3,4 at random, e.g.,

```
> R(), R(), R();
```

$$3, 3, 2 \tag{15.1}$$

Solution 8