

MACM 204 Makeup Assignment - Practice Final.

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You do not have to do this assignment.

It is meant to be a practice assignment, a warm up for the final exam.

However, if you got a bad mark on one of your assignments, or you missed an assignment, you may do this assignment and I'll replace your worst mark with your mark for this one.

Due Thursday December 5th at 1:30pm.

There are 8 questions. There are 80 marks.

I will mark it out of 60. So you don't need to do all questions to get full marks.

You can answer all questions (please do) and if you get more than 60 I will give you 60.

If you are not sure how to do something, please feel free to come and ask me for help.

Please attempt each question in a separate worksheet.

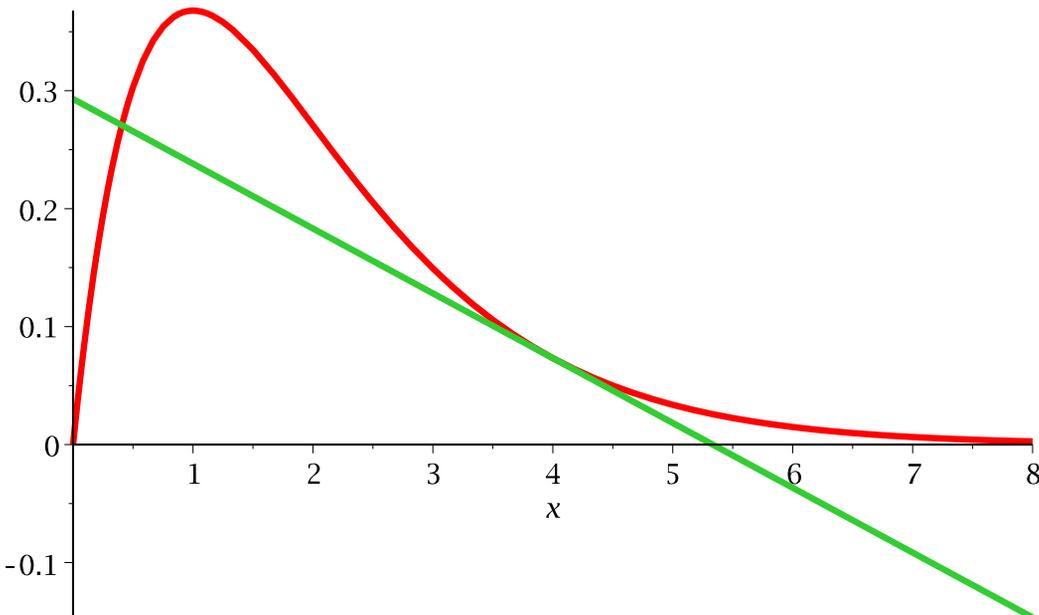
Print your Maple worksheets (double sided if you wish) and hand them in to me.

Question 1 (12 marks)

Shown in the figure below is a plot of $f(x) = xe^{-x}$ for $0 \leq x \leq 8$ and a graph of a tangent line T which is tangent to $f(x)$ at $x = 4$.

Compute the tangent line $T(x)$ and recreate the plot using the **plot** command.

Solve $T(x) = f(x)$ to find the intersection point near $x = 0.4$, $y = 0.27$ and compute the area enclosed between $f(x)$ and the tangent line to 5 decimal places (by computing a definite integral).



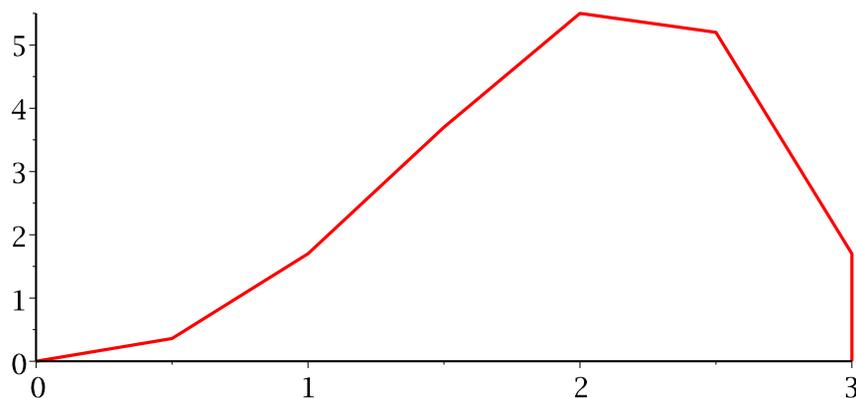
► Solution 1

▼ Question 2 (10 marks)

```
> Data := [[0.0,0.0], [0.50,0.36], [1.,1.7], [1.5,3.7],  
           [2.0,5.5], [2.5,5.2], [3.0,1.7], [3.0,0.0]];
```

```
Data:= [[0., 0.], [0.50, 0.36], [1., 1.7], [1.5, 3.7], [2.0, 5.5], [2.5, 5.2], [3.0, 1.7], [3.0, 0.]]
```

Below is a plot of the data points with lines between them.



First, recreate the plot of the data.

Second, write a Maple procedure **TrapezoidalRule(Data)** that on input of a list of data points, each point of the form $[x_i, y_i]$, computes the area of the trapezoids to approximate the area under the solid curve in the figure above. Test your Maple procedure on the data above. You should get 8.655.

► Solution 2

▼ Question 3 (10 marks)

Consider the function $f(x, y)$ below. Compute the formula for the tangent plane $T(x, y)$ at $x = 1, y = 1$. Generate a plot of $f(x, y)$ AND the tangent plane on the same plot using the **plot3d** command. Choose options for the tangent plane and $f(x, y)$ so that each is distinct and easily visible.

```
> f := 1-x^2-y^2-2*x;
```

$$f := -x^2 - y^2 - 2x + 1$$

Now on a second plot, graph the surface $f(x, y)$ and the curves $f(1, y)$ and $f(x, 1)$ on the same plot.

Use the **spacecurve** command from the **plots** package for the curves. Use $0 \leq x \leq 2$ and $0 \leq y \leq 2$ for the domain.

► Solution 3

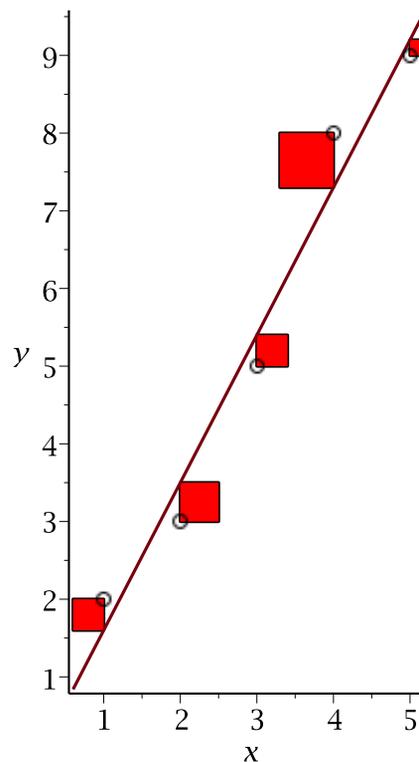
Question 4 (10 marks)

Suppose we have some $[x,y]$ data points

```
> Data := [[1,2],[2,3],[3,5],[4,8],[5,9]];
          Data:= [[1, 2], [2, 3], [3, 5], [4, 8], [5, 9]]
```

Suppose we approximate the data with best line using least squares

```
> with(Student):
> with(LinearAlgebra):
> infolevel[Student[LinearAlgebra]] := 2:
> LeastSquaresPlot( Data, [x,y], curve=a+b*x, boxoptions=[color=red] )
;
Fitting curve: -.3000+1.900*x
Least squares error: 1.049
Maximum error: .7000
```



Least-squares fit of the curve $y = bx + a$ to 5 given data points.

So according to the output, the line of least squares fit is $y = f(x)$ where $f(x) = -0.30 + 1.90 \cdot x$

Create the plot by graphing the line then creating red squares using `POLYGONS(...)` or the **rectangle** command in the `plottools` package. Don't worry about the points. So the first square goes from $[1, f(1)] = [1, 1.6]$ up to the first data point $[1, 2]$. I suggest you write a little procedure **MakeSquare**(A, B) that and takes as input the two points $A = [x, f(x)]$ and $B = [x, y]$ and outputs one square.

► Solution 4

▼ Question 5 (10 marks)

Consider linear systems of the following form.

Let A be an n by n matrix with 2 on the diagonal and 1 on the super diagonal and 1 on the sub diagonal and 0 else where. Let b is a vector of 1's and let x be the solution of the linear system $Ax = b$.

For example, for $n=6$ we have

$$A := \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \quad b := \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad x := \begin{bmatrix} \frac{3}{7} \\ \frac{1}{7} \\ \frac{2}{7} \\ \frac{2}{7} \\ \frac{1}{7} \\ \frac{3}{7} \end{bmatrix}$$

Now solve the linear systems for $n=4,5,6,7, \dots$ and determine a formula for the solutions. Use the **LinearSolve** command in the **LinearAlgebra** package. You will find that the formula depends on whether n is even or odd.

► Solution 5

▼ Question 6 (8 marks)

Do you remember Simpson's rule from MATH 152? It's a numerical integration rule. To approximate the definite integral $A = \int_a^b f(x) dx$ we let $h = \frac{(b-a)}{2}$ and compute $f(a), f(a+h), f(b = a+2 \cdot h)$. Simpson's rule fits (interpolates) the three points

$(a, f(a)), (a+h, f(a+h)), (a+2 \cdot h, f(a+2 \cdot h))$

with a quadratic $Q(x) = A \cdot x^2 + B \cdot x + C$. Then it approximates the definite integral A with

$S = \int_a^b Q(x) dx$ thus obtaining the formula for Simpson's rule namely

$$S = \frac{h}{3} \cdot (f(a) + 4 \cdot f(a+h) + f(a+2 \cdot h)).$$

Note this approximation is much more accurate than the Trapezoidal rule

$$T = \frac{h}{2} \cdot (f(a) + 2 \cdot f(a+h) + f(a+2 \cdot h)).$$

Use Maple to derive the formula for Simpson's rule.

Step 1: Find $Q(x)$ using data values $(a, f(a)), (a+h, f(a+2\cdot h)), (a+2\cdot h, f(a+2\cdot h))$. Do this by setting up a linear system of equations to solve for A, B, C .

Step 2: Once you have a formula for $Q(x)$ just use Maple to compute $S := \int_a^b Q(x) dx$. To

get Simpson's formula you'll need to simplify the result. If you are not sure what is going on, sketch a function $f(x)$, mark the points $x = a, a+h, b$ and $f(a), f(a+h), f(b)$, then sketch $Q(x)$ and shade the area S . Then try to come up with $Q(x)$ by hand and use Maple to solve the linear system.

► Solution 6

▼ Question 7 (8 marks)

For $n = 1, 2, 3, \dots$ let $P(n)$ be the number of primes $\leq n$.

So that $P(1) = 0, P(2) = 1, P(3) = 2, P(4) = 2, P(5) = 3$, etc.

Write Maple procedure **PRIMES** that returns a Maple Array P of length n such that

$P[1] = 0, P[2] = 1, P[3] = 2, P[4] = 2, P[5] = 3$ etc.

Your procedure should initialize **P := Array(1..n)**;

Use the **isprime** command to test for primality.

Test your procedure on **PRIME(30)**

► Solution 7

▼ Question 8 (12 marks)

Consider the function $f(x) = \frac{\sin(x)}{x}$.

Graph $y = f(x)$ and the line $y = x$ on the same plot on the domain $0 \leq x \leq 2$.

Solve $f(x) = x$ to 5 decimal places.

Calculate $f'(1)$ to 5 decimal places.

Evaluate $\int_0^1 f(x) dx$ to 5 decimal places.

Expand $f(x)$ as a Taylor series to $O(x^{10})$.

Calculate the $\lim_{x \rightarrow 0} f(x)$ in Maple.

► Solution 8