

Algorithm  $p$ -adic  $\sqrt{\phantom{x}}$  ( $a, u_0, p, B$ )

Input  $a \in \mathbb{Z}^+$

$p > 2$

$u_0 \in \mathbb{Z}$  s.t.  $a - u_0^2 \equiv 0 \pmod{p}$

and  $u_0 \not\equiv 0 \pmod{p}$

$B > \sqrt{a}$  a bound.

Output FAIL  $\Rightarrow \sqrt{a} \notin \mathbb{Z}$  or  $\sqrt{a}$

$u \leftarrow \text{mods}(u_0, p)$

$i \leftarrow 1/(2u_0) \pmod{p}$

for  $k=1, 2, 3, \dots$  do

$e \leftarrow a - u^2$

if  $e=0$  then output  $u$ .

if  $p^k > 2B$  then output FAIL

$e \leftarrow e/p^k$

$u_k \leftarrow \text{mods}(i \cdot e, p)$

$u \leftarrow u + u_k p^k$

end for

end.

end.