

Compute a sqrt in  $\mathbb{Z}$  using a linear  $p$ -adic Newton iteration.

```
> NI := proc(u0::integer,a::posint,p::prime,B::posint) local u,e,uk,
pk,k,i;
  u := mods(u0,p);
  i := (2*u0)^(-1) mod p;
  pk := p;
  k := 1;
  while true do
    e := a-u^2;
    if e=0 then return u; fi;
    if pk > 2*B then return FAIL; fi;
    e := iquo(e,pk) mod p;
    uk := mods(i*e,p);
    u := u + uk*pk;
    pk := p*pk;
  od;
end;
> a := 131^2;
                                     a := 17161
> p := 7;
                                     p := 7
> Factor( x^2-a ) mod p;
                                     (x + 5) (x + 2)
> NI(-2,a,p,200);
                                     131
> NI(2,a,p,200);
                                     -131
> p := prevprime(10^4):
> a := 3^20000: u0 := 3^10000 mod p:
> time(NI(u0,a,p,a));
                                     0.109
> a := a*a: u0 := u0*u0 mod p:
> time(NI(u0,a,p,a));
                                     0.380
> a := a*a: u0 := u0*u0 mod p:
> time(NI(u0,a,p,a));
                                     2.300
> a := a*a: u0 := u0*u0 mod p:
> time(NI(u0,a,p,a));
                                     11.452
```