Theorem 12.7 (Trager-Rothstein - Logarithmic case)

$\int \frac{C(\theta)}{D(\theta)} dx$ $\quad C, D \in F[\theta]$, $\theta = \log u$, $u \in F$, $\theta \notin F$, $\theta' \neq 0$
$0 \leq \deg_\theta C < \deg_\theta D$, $lc_\theta D = 1$,
$\gcd(C, D) = 1$, $\gcd(D, D') = 1$ in $F[\theta]$.

Let $R(z) = \mathrm{res}_\theta(C - z\underset{\overset{\cap}{F[\theta]}}{D'}, D) \in F[z]$

(i) $\int \frac{C}{D} dx$ is elementary iff all roots of $R(z)$ are constants

(ii) If $\int \frac{C}{D} dx$ is elementary then $\int \frac{C}{D} dx = \sum c_i \log v_i$

where c_i are the distinct roots of $R(z)$
and $v_i = \text{monic } \gcd(C - c_i D', D) \in F[\theta]$