$$\int p_\ell\theta^\ell + \cdots + p_0 = \underset{\in K}{q_{\ell+1}}\,\theta^{\ell+1} + \underset{\in F}{q_\ell}\,\theta^\ell + \cdots + \underset{\in F}{q_1}\,\theta + \underset{\in F}{q_0} + \Sigma L$$

Differentiating and equating coefficients in θ^i yields

in θ^ℓ	$p_\ell = (\ell+1)\,q_{\ell+1}\theta' + q_\ell'$	(ℓ)
in $\theta^{\ell-1}$	$p_{\ell-1} = \ell q_\ell\theta' + q_{\ell-1}'$	$(\ell-1)$
$\vdots$		
in θ^1	$p_1 = 2q_2\theta' + q_1'$	(1)
in θ^0	$p_0 = q_1\theta' + q_0' + \Sigma L'$	(0)

Integrating both sides of (ℓ) yields

$$\int \underset{\in F}{p_\ell} = \underset{\in K}{\underline{(\ell+1)q_{\ell+1}}}\,\theta + \underset{\in F}{q_\ell}$$

[Liouville $\Rightarrow$ if $\int p$ is elementary then $\int p_\ell$ is of this form]

Compute $\int p_\ell$ recursively in F.
If $\int p_\ell$ is not elementary then $\int p$ is not elementary.
If $\int p_\ell$ is elementary and $\int p_\ell = c\log v + \cdots$ and $\log v \notin F(\theta$
then $\int p_\ell$ is not elementary.

Otherwise $\underset{\text{Liouville}}{\Rightarrow} \int p_\ell = \underset{\in F}{v_\ell} + \underset{\in K}{b_\ell} + c_\ell\theta = (\ell+1)q_{\ell+1}\theta + q_\ell$

Solving for $q_{\ell+1}, q_\ell \Rightarrow q_{\ell+1} = c_\ell/\ell+1$ and $q_\ell = v_\ell + b_\ell$
Substitute $q_\ell = \underset{\in F}{v_\ell} + \underset{\in K}{b_\ell}$ into $(\ell-1)$ yields

$$p_{\ell-1} = \ell(v_\ell + b_\ell)\theta' + q_{\ell-1}'$$

$$\Rightarrow p_{\ell-1} - \ell v_\ell\theta' = \ell b_\ell\theta' + q_{\ell-1}'$$

$$\Rightarrow \int \underline{p_{\ell-1} - \ell v_\ell\theta'} = \ell b_\ell\theta + q_{\ell-1} \qquad \textit{Repeat!}$$