

Example Hermite Reduction (Exponential Subcase).

$$\int \frac{e^x + 1}{e^{2x} + 2xe^x + x^2} = \int \frac{\theta + 1}{\theta^2 + 2x\theta + x^2} = \int \frac{\theta + 1}{(\theta + x)^2}$$

$F(\theta) = Q(\theta)(e^x)$ $\xrightarrow{\text{Square Free-factorization}}$

$$\int \frac{\theta + 1}{(\theta + x)^2} = \int \frac{P}{Q = q_1 q_2^2 \dots q_k^k} \quad \begin{matrix} k=2 & q_1=1 & q_2=\theta+x \\ T=Q/q_k^k=1 & q_2'=\theta+1 \end{matrix}$$

Solve $\sigma q_k' T + \tau q_k = P$ for $\sigma, \tau \in F[\theta]$.
 $\Rightarrow \sigma(\theta+1) + \tau(\theta+x) = \theta+1 \Rightarrow \sigma=1, \tau=0$

$$\int \frac{P}{Q = q_1 q_2^2 \dots q_k^k} = \frac{-\sigma/(k-1)}{q_k^{k-1}} + \int \frac{\tau + \sigma/(k-1)T}{Q/q_k}$$

$$\int \frac{\theta+1}{(\theta+x)^2} = \frac{-1/1}{\theta+x} + \int \frac{0 + 0/1 \cdot 1}{\theta+x} = \frac{-1}{e^x + 2x}$$