Chapter 5 Homomorphisms and Chinese Remainder Algorithms

5.3 Ring Morphisms

Let R and S be two rings with identities 1_R and 1_S. A function $\phi : R \to S$ is called a ring morphism (or homomorphism) if $\forall a, b \in R$

(i) $\phi(a +_R b) = \phi(a) +_S \phi(b)$
(ii) $\phi(a \cdot_R b) = \phi(a) \cdot_S \phi(b)$ and
(iii) $\phi(1_R) = 1_S$

Lemma. Let $a \in R$. Then

(iv) $\phi(0_R) = 0_S$
(v) $\phi(-a) = -\phi(a)$
(vi) a is a unit $\Rightarrow \phi(a)$ is a unit

Proof (iv)