

Some examples of Linear Algebra in Maple

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Vector input and arithmetic

```
> u := <1,1>;
```

$$u := \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (1)$$

```
> v := Vector([2,1]);
```

$$v := \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad (2)$$

```
> u.v; # dot product
```

$$3 \quad (3)$$

```
> u+v;
```

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad (4)$$

Matrix input and arithmetic

```
> A := Matrix([[2,1],[1,2]]);
```

$$A := \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (5)$$

```
> B := Matrix([[1,1],[1,0]]);
```

$$B := \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \quad (6)$$

```
> A+B;
```

$$\begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix} \quad (7)$$

```
> A.B;
```

$$\begin{bmatrix} 3 & 2 \\ 3 & 1 \end{bmatrix} \quad (8)$$

```
> A.u;
```

$$\begin{bmatrix} 3 \\ 3 \end{bmatrix} \quad (9)$$

```
> (A^2).u;
```

$$\begin{bmatrix} 9 \\ 9 \end{bmatrix} \quad (10)$$

```
> A.(A.u);
```

(11)

$$\begin{bmatrix} 9 \\ 9 \end{bmatrix} \quad (11)$$

Solving $A \cdot x = b$

> **b := <1,1>;**

$$b := \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (12)$$

> **Ab := <A|b>;**

$$Ab := \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \quad (13)$$

> **with(LinearAlgebra):**

> **R := ReducedRowEchelonForm(Ab);**

$$R := \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} \end{bmatrix} \quad (14)$$

> **R[1..2,3];**

$$\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \quad (15)$$

> **LinearSolve(A,b);**

$$\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \quad (16)$$

Matrix inverse

> **I2 := <<1,0>|<0,1>>;**

$$I2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (17)$$

> **B := <A|I2>;**

$$B := \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \quad (18)$$

> **R := ReducedRowEchelonForm(B);**

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$$R := \begin{bmatrix} 1 & 0 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \quad (19)$$

> R[1..2,3..4];

$$\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \quad (20)$$

> 1/A;

$$\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \quad (21)$$

Determinants and characteristic polynomials

> A;

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (22)$$

> Determinant(A);

$$3 \quad (23)$$

> CharacteristicPolynomial(A,x);

$$x^2 - 4x + 3 \quad (24)$$

> x*I2-A;

$$\begin{bmatrix} x-2 & -1 \\ -1 & x-2 \end{bmatrix} \quad (25)$$

> Determinant(x*I2-A);

$$x^2 - 4x + 3 \quad (26)$$

Matrix input using a loop

> T4 := Matrix(4,4);

$$T4 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (27)$$

```
> for i to 4 do
  for j to 4 do
    T4[i,j] := abs(i-j);
  od;
od;
```

```
od;
T4;
```

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

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```
> C := [x[1],x[2],x[3],x[4]];
for i to 4 do
  for j to 4 do
    T4[i,j] := C[ abs(i-j)+1 ];
  od;
od;
T4;
```

$$C := [x_1, x_2, x_3, x_4]$$

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2 & x_1 & x_2 & x_3 \\ x_3 & x_2 & x_1 & x_2 \\ x_4 & x_3 & x_2 & x_1 \end{bmatrix}$$

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```
> Determinant(T4);
```

$$x_1^4 - 3x_1^2x_2^2 - 2x_1^2x_3^2 - x_1^2x_4^2 + 4x_1x_2^2x_3 + 4x_1x_2x_3x_4 + x_2^4 - 2x_2^3x_4 - 2x_2^2x_3^2 + x_2^2x_4^2 - 2x_2x_3^2x_4 + x_3^4$$

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