Integrals and series by computer algebra

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LETTERS TO THE EDITOR

Letters are selected for their expected interest for our readers. Some letters are sent to reviewers for advice; some are accepted or declined by the editor without review. Letters must be brief and may be edited, subject to the author's approval of significant changes. Although some comments on published articles and notes may be appropriate as letters, most such comments are reviewed according to a special procedure and appear, if accepted, in the Notes and Discussions section. (See the "Statement of Editorial Policy" in the January issue.) Running controversies among letter writers will not be published.

INTEGRALS AND SERIES BY COMPUTER ALGEBRA

In his comparative review of *Integrals and Series* and other tables,[1] Romer has provided a useful (if not comprehensive) guide to the reference literature for analytic expressions of definite and indefinite integrals, series, and other mathematical functions.

However, on reading this review, I was immediately struck by the feeling that an important aspect of the application of such tables has been overlooked. Imagine—if you can, but really it is unthinkable—a review in 1988 of tables of logarithms in which the author neglected to state that for a paltry sum of money one could purchase a pocket calculator that can in practice supply far more accurate values, and far more rapidly, of not only common (Briggsian) but also natural (Napierian) logarithms than any known table. Really an analogous situation applies, to a much greater extent than might have been supposed, to much content of the reviewed tables, through computer algebra (also called symbolic computation).

For instance, by means of the processor DERIVE recently marketed (at a cost about half that of the most expensive set of volumes in the review) for use on the almost ubiquitous PC-compatible microcomputers, I obtained the solution of the integral for the specified integrand $(ax^2 + bx + c)^{-1/2}$ in 0.3 s of CPU time, and even more quickly verified the answer by differentiation. This processor also generates automatically two- or three-dimensional plots, or parametric plots, and the motion of the cursor around the plotting surface can determine the coordinates of any point on the screen with reference to the displayed system of axes. Other processors for computer algebra, such as MAPLE, MUMATH, or REDUCE, have varying but broadly overlapping capabilities, operating on machines from popular microcomputers to supercomputers.

In my article,[2] besides a discussion of applications in physics education, a brief description of the available processors and a summary of their exemplary applications across the breadth of physics are included. Not only the "research" universities but really any university, college, or even secondary school should consider as a matter of urgency the implementation of a Mathematical Laboratory that could give their students (and teachers) access, by means of computers, to analytic operations in mathematics, algebra, calculus, geometry, trigonometry, group theory, etc., in much the same way that the pocket calculator has already done for arithmetic and numerical functions.

Any future reviewer of such printed tables of integrals or functions would indeed be remiss to fail to refer to an entirely mature and viable alternative method of yielding the required analytic solution in a significant proportion of the cases. The method of computer algebra is truly effective in terms of time and cost, and educators and their students need no longer rely (entirely) on either manual derivations or printed matter for their analytic applications.

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[1] Robert H. Romer, Am. J. Phys. **56**, 957–959 (1988).
[2] J. F. Ogilvie, "Computer Algebra in Modern Physics," Comput. Phys. **3**, 66–74 (1989).

ARE DELIBERATE MISTAKES A VALID TEACHING TOOL?

It is with some trepidation that I write concerning the problem presented by Crawford.[1] He states a problem of an initially nearly vertical stick with the lower end resting on a perfectly frictionless table. He presents a solution, then tells the reader to think about the result, and finally asks, "Where did we goof?" Then, on another page, he reveals he improperly took moments/torques about a moving point (the contact point).

The impropriety of taking moments about a moving point is often forgotten and is often poorly discussed in elementary texts. It is most important that students *know* and *understand the reasons* that moments may only be taken about either a fixed point, or the c.m., or about an instantaneously stationary point. So Crawford's point is an excellent one, worthy of repetition.

Yet I cringe when I see this deliberate mistake in print, regardless of the "good" motive—even when in mitigation we are first warned to "look well," and shortly thereafter all is revealed to us. To compound the injury, Crawford makes no attempt to *explain* why taking moments about a moving point can be disastrous.

Actually, in part, his intent is to consider and discuss the correct solution to his problem. I find that this latter purpose gets lost in the confusion he engenders by his deliberate mistake.

Personally, I was well taught and am very sensitive to this possible error. Even though Crawford did not catch me with his printed "deliberate mistake," I still felt professionally mistreated. Am I too squeamish?

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[1] Frank S. Crawford, "Problem: Moments to Remember," Am. J. Phys. **57**, 105, 177 (1989).