A New Polynomial Data Structure For Maple

SFU

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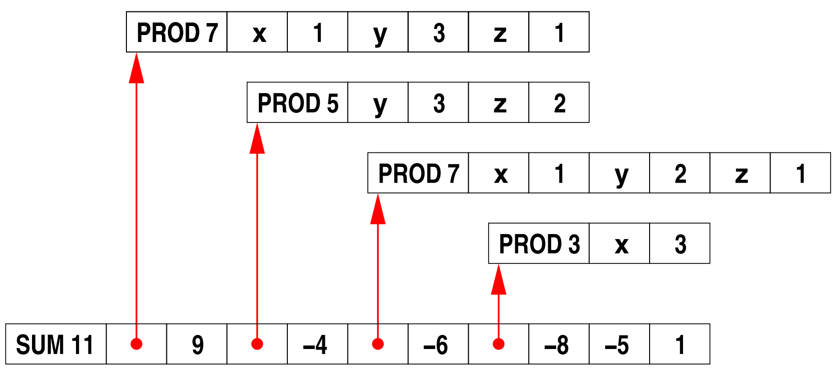
Polynomial Representation

Maple's current representation for polynomials is a *sparse sum of products:*

$$9\,xy^3z - 4\,y^3z^2 - 6\,xy^2z - 8\,x^3 - 5$$



This is slow for large polynomials because:

- common operations must examine every term (e.g. degree, set of variables, type checks)
- each monomial adds overhead to the system
- monomials are spread out all over memory
- monomial operations are complicated

Maple also sorts polynomials by monomial address, so whenever monomials are changed it must re-sort.

Overhead of Maple's Representation

Multiply $f = (1 + x + y + z)^{20}$ and $g = f + 1$

Total time: 0.028 sec

type check		
+ get variables		
+ compute degree	0.005 sec	(18%)
call sdmp C library	0.014 sec	(50%)
build Maple result	0.009 sec	(32%)

On sparse problems (and dense problems with dense algorithms) the overhead can be over 97%.

Packed Monomials

Our software (sdmp) uses a packed distributed format to achieve high performance. Monomials are represented as machine integers.

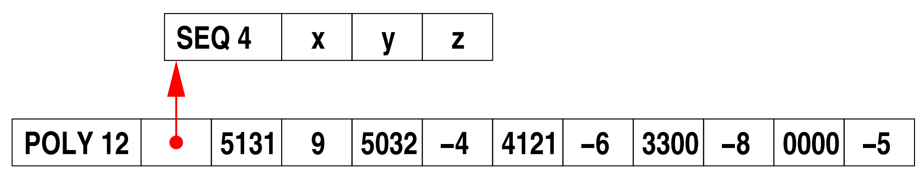
$x^3y^2z^1$	$\Longrightarrow$	$[6\;3\;2\;1]$	$\Longrightarrow$	00000110 00000011 00000010 00000001
degree: 6		exponents		bits on a 32-bit computer

- Monomial multiplication adds machine integers in C
- To divide monomials, we subtract and check for underflow
- Term ordering uses unsigned integer comparisons

Poly DAG

Polynomials with integer coefficients have a *new dag:*

$$9\,xy^3z - 4\,y^3z^2 - 6\,xy^2z - 8\,x^3 - 5$$



It uses *graded lexicographical order*. Polynomials will appear sorted.

The maximum total degree is determined by the number of variables:

# variables	32-bit max	64-bit max
2	1023	2097151
3	255	65535
4	64	4095
5	31	1023
6	15	511
7	15	255
8	7	127

Many operations go from $O(n) \longrightarrow O(1)$:

- $indets(f)$ and $has(f, x)$ look at the variables
- $degree(f)$ and $lcoeff(f)$ look at the first term
- $expand(f), normal(f), numer(f), denom(f)$ do nothing
- $type(f, polynom)$ knows it is a polynomial over $\mathbb{Z}$

Overhead is **20x lower** with this new data structure.

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