Overview

The structure and information of a group, G, can be represented in many ways. One can compute its entire multiplication table, also known as its Cayley Table, or determine all the subgroups of G and arrange them in a lattice, or identify isomorphic forms of the same group. We have implemented some new prospective additions for the group and the upcoming FiniteGroups package of Maple.

New representations for groups have also been added. Well known families of groups such as the Alternating Group or specific groups such as the Tetrahedral group can be called by name in symbol format. In addition, there is a matrix representation for groups in the form of MatrixGroup(p, S). The group is formed with a set of generators S under multiplication modulo p. If p = 0, then we are multiplying normally.

Cayley Table

Given a set of generators S for a group G, DrawCayleyTable draws the group’s Cayley Table using Dimino’s Algorithm to generate all the elements of G. Each element is associated with a distinct colour and label. There are various labelling, ordering, and colouring options for every element.

With multiple subgroups H1, · · · , Hn, one can highlight the elements of each subgroup Hj in the Cayley Table. Elements in the same subgroup are coloured identically, but if an element, such as the identity, is in more than one subgroup, the element’s colour is blended. Note that a subgroup can either be specified in a group format or as a set of generators.

Isomorphism Test

The isomorphism problem is an exponentially hard problem. Hence, finding the best invariants to distinguish two groups is crucial. Our computational experiments strongly suggest that the conjugacy classes and commutativity relations in a group are good tests for isomorphism. Using these invariants (and a few others), the Isomorphism procedure can determine whether two groups G1, G2 are isomorphic.

If G1 ∼ G2, the procedure returns true, along with an explanatory message.

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Visualizing Groups in Maple

Subgroup Lattice

If G is a group, then its subgroup lattice, L(G) = (V, E), is defined as follows:

○ The vertex set V is composed of all distinct subgroups H in G.

○ If [H] is a product of exactly i primes, then H is in level i of L(G), denoted H ∈ L(i)(G).

○ Suppose H ∈ L(i)(G) and K ∈ L(j)(G) where j > i. Then [H, K] ∈ E if and only if H ⊂ K and j − i is minimal.

○ If [H, K] ∈ E and |H| < |K|, then its edge weight is defined as the index between the two subgroups |H|/|K|.

Given a solvable group G, the procedure DrawSubgroupLattice can display the complete L(G) using the Cyclic Extension Method. DrawSubgroupLattice returns the plot or graph of L(G), and the list of elements of G. Each vertex is labelled with the generators of its group; the generators are labelled by the corresponding index in the returned list of elements, below defined as S.

Future Projects

○ Inclusion of all perfect subgroups in DrawSubgroupLattice

○ Draw the Cayley Graph given a set of generators for a group

○ Efficient group representation for Matrices over a Galois Field

○ Highlighting of special subgroups in L(G), such as normal ones.

Special thanks to Greg Fee, Kseniya Garaschuk, and Paul Vrbik for their help and advice.