

SIMON FRASER UNIVERSITY

**MATH 320 - Introduction to  
Analysis II Midterm**

**February 24, 2006, Friday,  
13:30 – 14:20**

Last Name _____
Given Name(s) _____
Student # _____

Student signature
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**INSTRUCTIONS**

1. Write your last name, given name(s), and student number in the box above.
2. Sign your name in the box provided.
3. This exam has 3 questions on 5 pages. Please check to make sure your exam is complete.
4. This is a closed book exam.
5. Calculator is allowed.
6. Ask for clarification if you cannot understand a question or there appears to be an error.
7. Use the reverse side of the previous page if you need more room for your answer.
8. **You may lose marks if your explanations are incomplete or poorly presented.**

Question	Maximum	Score
1	30	
2	40	
3	30	
<b>Total</b>	<b>100</b>	

[30] 1. Let  $A_1, A_2, A_3, \dots$  be subsets of a metric space.

(a) If  $B_n = \cup_{i=1}^n A_i$ , prove that  $\overline{B_n} = \cup_{i=1}^n \overline{A_i}$ , for  $n = 1, 2, 3, \dots$ .

(b) If  $B = \cup_{i=1}^{\infty} A_i$ , prove that  $\overline{B} \supset \cup_{i=1}^{\infty} \overline{A_i}$ .

Show, by an example, that this inclusion can be proper.

- [15] 2. (a) Let  $f : X \longrightarrow Y$  be a function from metric spaces  $(X, d_1)$  into  $(Y, d_2)$ . Prove that if  $f$  is continuous on  $X$ , then  $f^{-1}(U)$  is open in  $X$  for every open subset  $U$  in  $Y$ .

[5] (b) Let  $f : X \longrightarrow Y$  be a function from metric spaces  $(X, d_1)$  into  $(Y, d_2)$ . Give definition of **uniformly continuous functions** of  $X$ .

[20] (c) Let  $f : X \longrightarrow Y$  be a continuous function of  $X$ . Let  $X$  be a compact space. Show that  $f(X)$  is also compact.

[10] 3. (a) Let  $(X, d)$  be a metric space. Fix  $p \in X, \delta > 0$ , define

$$A := \{q \in X : d(p, q) < \delta\}$$

and

$$B := \{q \in X : d(p, q) > \delta\}.$$

Prove that  $A$  and  $B$  are separated.

- [20] (b) Hence prove that every connected metric space with at least two points is uncountable.

**END**