

Assignment 1, MACM 204, Fall 2014

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The purpose of this assignment is to learn the basics of Maple: how to input formulas, how to graph functions, solve equations, calculate derivatives and integrals, and to use various kinds of loops (i.e. some basic programming).

Due Tuesday September 17th at 4:00pm in the drop off box.

Do all calculations in Maple. Please print your Maple worksheets.

You may compress the plots to save paper.

Question 1

Consider the function $f(x) = x e^{-x}$. Construct a plot of $[f(x), f'(x), f''(x)]$ on the same graph for the domain $0 \leq x < 10$.

Read the help page for `?plot,options`. Using the appropriate options, create a legend for the plot, and add a title.

Question 2

Calculate the following values to **20 decimal places**

$$\pi = 3.141\dots, \quad e^2 = 7.389\dots, \quad \sqrt{2} = 1.414\dots, \quad \ln(2) = 0.693\dots, \quad \text{and} \quad \int_0^1 \sin(x^2) dx = 0.310\dots$$

Question 3

Consider the polynomials $f := x^4 - 1$ and $g := x^4 - 4x^3 + 8x - 4$ and $h := x^4 - 4x^3 + 4x^2 - 8x + 4$.

Graph each separately on a suitable domain for x so that we can see all real roots.

Now factor the polynomials. Why does the factor command not factor them into linear factors?

Now, using `solve`, solve for the roots. Verify that the 4 roots of the polynomial g are correct.

Now, using `fsolve`, solve for numerical approximations of the roots, including the complex roots.

Read the help page for `?fsolve` to find out how to get the complex roots.

Question 4

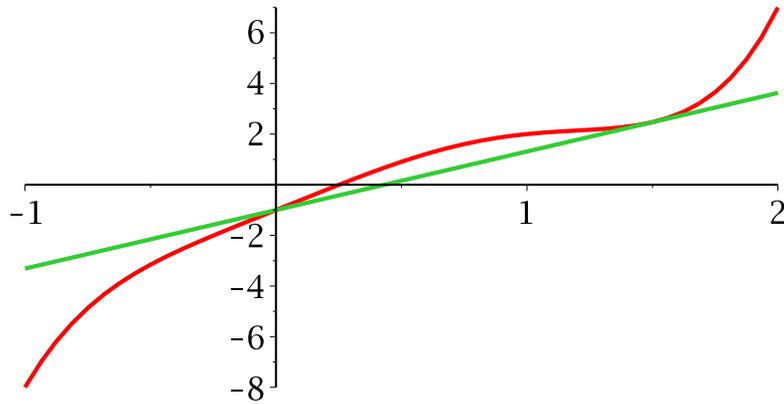
Shown in the plot below is a plot of $f := x^5 - 2x^4 + 4x - 1$ on the domain $-1 < x < 2$.

Shown also is the line tangent to $f(x)$ at $x = \frac{3}{2}$. Use Maple to reproduce the plot. Now

express the area enclosed by the tangent line and $f(x)$ as a definite integral. Evaluate the definite integral using Maple.

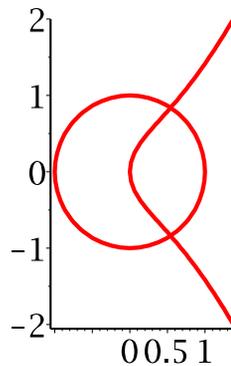
```
> f := x^5 - 2*x^4 + 4*x - 1;
```

$$f := x^5 - 2x^4 + 4x - 1$$



Question 5

Consider the two equations $y^2 = x^3 + x$ and $x^2 + y^2 = 1$ which are the equations of an elliptic curve and a circle. Graph the two curves together using the `plots[implicitplot]` command. Improve the smoothness of the plot using the `grid` option if necessary. The circle will appear distorted. Find out from `?plot,options` how to scale the axes so that the circle looks like a circle (so that the plot looks like the figure below). You can read off from the plot roughly where the curves intersect. Using the `fsolve` command compute the intersection points accurately.



Question 6

Calculate the following antiderivatives in Maple.

(i) $\int x^2 e^{-x} dx$ (ii) $\int x^2 \cos(x) dx$ (iii) $\int \sqrt{x} \cos(x) dx$.

To check that the results are correct, use Maple to differentiate each antiderivative. Now try to find an antiderivative that Maple cannot do.

The last four questions are to get you to write some loops and to experiment.

Question 7

Consider the definite integral $\int_0^{\infty} x^n e^{-x} dx$.

For $n = 1, 2, 3, 4, 5$ use Maple to calculate the integrals in a loop.
From these values deduce the formula for the integral as a function of n .

Question 8

The first n terms of the Taylor series for e^x are

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots + \frac{x^n}{n!}.$$

Write a Maple **for loop** that constructs the Taylor polynomial for any value of n . Run it for $n = 10$.

Question 9

The so called **3 x + 1** conjecture is the following.

If you start from any positive integer x , e.g., $x=9$, and you apply the following rule repeatedly

if x is even, divide x by 2 else multiply x by 3 and add 1

you get 9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, ...

You see that we end up at 1 then we get into a little cycle.

The conjecture is that for every positive integer x you ALWAYS get to 1. No one has proven it though!

Write a Maple **while loop** that generates the sequence for different values of x to get a feel for what happens. Try to find a value of x less than 50 that has a very long sequence.

Question 10

Let $A = \int_a^b f(x) dx$. Recall that the value of A may be approximated by the Trapezoidal

rule on n intervals of width $h = \frac{(b-a)}{n}$ using the formula

$$T_n = \frac{h}{2} \cdot (f(a) + 2 \cdot f(a+h) + 2 \cdot f(a+2 \cdot h) + \dots + 2 \cdot f(a+(n-1) \cdot h) + f(b)).$$

For $f(x) = x^2 \cdot \sin(2 \cdot x)$ and $a = 0, b = 1$, calculate A using Maple to 10 decimal places by using Maple's integration command.

Now calculate T_8, T_{16} and T_{32} to 10 decimal places in a loop. How accurate is T_{32} ?