Assignment 3, MACM 204, Fall 2016

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Due Tuesday October 25tht at 3:00pm.
Late penalty: -20% for up to 48 hours late. 0 after that.

Please attempt each question in a seperate worksheet (so that you don't destroy your previous work). Print your Maple worksheets (you may print double sided if you wish).

Question 1

Part (a)
Solve the following linear systems in Maple using the solve command.
For each system, how many solutions are there? (you have to interpret the output from Maple).

```
> sys1 := [x+y+z=2, x-y=1, x+z=0];
```

$$sys1 := [x+y+z=2, x-y=1, x+z=0]$$

```
> sys2 := [x+y+z=2, x-y=1, 2*y+z=1];
```

$$sys2 := [x+y+z=2, x-y=1, 2\,y+z=1]$$

```
> sys3 := [x+y+z=2, x-y=1, 2*y+z=-1];
```

$$sys3 := [x+y+z=2, x-y=1, 2\,y+z=-1]$$

Part (b)
Recall that an equation of the form $a \cdot x + b \cdot y + c \cdot z = d$ is the equation of a plane in 3 dimensions.
Graph each system (of three planes) on the same plot using the implicitplot3d command for a suitable domain. Give the three planes different colours. Rotate the plot so that we can see the main solution(s) from solve.

Question 2

Consider the 3 by 3 system of polynomial equations

$$\{x^2+y^2+z^2=4, x^2+y^2-z^2=4, x+y+z=0\}$$

Part (a). Solve the system using the solve command. You will get solutions involving RootOf($\_Z^2-2$) which stands for $\pm\sqrt{2}$. To get solutions in terms of $\sqrt{2}$ use this option before you call solve.

```
> _EnvExplicit := true;
```

Part (b). Solve the system with the fsolve command. You may not get any solutions. Find out how to give fsolve an initial approximation of the solutions so that it will converge to the solutions.

Question 3

Consider the function

```
> f := 4*y^3+x^2-12*y^2-36*y+x^3+2;
```

$$4\,y^3+x^2-12\,y^2-36\,y+x^3+2$$

Find the critical points using Maple. You should get 4 of them. Determine which are saddle points, which are local minimums and which are local maximums.

Graph the surface $f(x, y)$ and highlight the critical points by drawing a spike (vertical line) through them using the **spacecurve** command in the plots package. Use blue for the saddles and red for the others so we can tell which is which.

Question 4

In question 9 of assignment 1 we were going through the odd primes 3,5,7,11,13,... counting the number of primes which are 1 mod 4 in S and the number of primes which are 3 mod 4 in T. I asked you to determine when |S| becomes larger than |T|. Here is my solution.

```
> S := 0;
  T := 0;
  p := 3;
  while p < 10^6 do
      if S=T and p mod 4 = 1 then
          printf("|S| > |T| for prime p=%d\n",p);
      fi;
      if p mod 4 = 1 then S := S+1; else T := T+1; fi;
      p := nextprime(p);
  od:
```

$$S := 0$$
$$T := 0$$
$$p := 3$$

```
|S| > |T| for prime p=26861
|S| > |T| for prime p=616841
|S| > |T| for prime p=616849
|S| > |T| for prime p=616877
|S| > |T| for prime p=617269
|S| > |T| for prime p=617369
|S| > |T| for prime p=617401
|S| > |T| for prime p=617429
|S| > |T| for prime p=617453
|S| > |T| for prime p=617521
|S| > |T| for prime p=617537
|S| > |T| for prime p=617689
|S| > |T| for prime p=617717
|S| > |T| for prime p=622813
|S| > |T| for prime p=623209
|S| > |T| for prime p=623321
|S| > |T| for prime p=623341
|S| > |T| for prime p=623353
|S| > |T| for prime p=623401
|S| > |T| for prime p=623437
|S| > |T| for prime p=623933
|S| > |T| for prime p=623957
|S| > |T| for prime p=623977
|S| > |T| for prime p=623989
|S| > |T| for prime p=624097
|S| > |T| for prime p=624133
|S| > |T| for prime p=624149
|S| > |T| for prime p=624241
|S| > |T| for prime p=626929
|S| > |T| for prime p=626953
```

```
|S| > |T| for prime p=627353
|S| > |T| for prime p=627449
|S| > |T| for prime p=627481
|S| > |T| for prime p=627733
|S| > |T| for prime p=627841
|S| > |T| for prime p=627901
|S| > |T| for prime p=628013
|S| > |T| for prime p=628937
|S| > |T| for prime p=628973
|S| > |T| for prime p=629341
|S| > |T| for prime p=629429
|S| > |T| for prime p=629513
|S| > |T| for prime p=630737
|S| > |T| for prime p=632813
|S| > |T| for prime p=632897
|S| > |T| for prime p=633013
|S| > |T| for prime p=633133
|S| > |T| for prime p=633161
|S| > |T| for prime p=633197
|S| > |T| for prime p=633449
|S| > |T| for prime p=633469
|S| > |T| for prime p=633649
|S| > |T| for prime p=633797
```

So up till $p=26861$ $|S| \leq |T|$. Modify my loop to compute how often |S| > |T| for odd primes < 1,000,000.
At the end of the loop, using the **printf** command, print out a message like this.

|S| > |T| only 0.031% of the time.

Question 5

Definition: Two functions $f(x)$ and $g(x)$ are orthogonal on $a \leq x \leq b$ if $\int_a^b f(x) \cdot g(x)\, \mathrm{d}x = 0$.

Example: $f(x) = 2 \cdot x^2 - 1$ and $g(x) = 4 \cdot x^3 - 3 \cdot x$ are orthogonal on $-1 \leq x \leq 1$.

You are to reproduce what I wrote above. You will need to input mathematics in a text region.
You can use the button labelled "Math" to go into math input mode and the button "Text" to go back to text input mode. Alternatively you can use <Ctrl> r to into math input mode and <Ctrl> t goes back to text input mode. To get a definite integral use the Expression pallette.

Finally, check that the example is correct using Maple.

Question 6

Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and vectors $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Part (a) Input A, u and v into Maple and calculate $2 \cdot u + w$, A^3 and A^{-1} .

Part (b) Using commands in the **LinearAlgebra** package calculate the determinant of *A*, the characteristic polynomial of *A* in the variable *x* and solve the linear systems $A \cdot x = u$ and $A \cdot x = w$ for x. Note, to load the package and see all the commands in the package do

```
> with(LinearAlgebra):
```

```
> ?LinearAlgebra
```

Part (c) Starting with the vector $v_1 = [1, 0]$ compute, in a for loop, the vectors

$v_2 = A \cdot v_1,\ v_3 = A \cdot v_2, v_4 = A \cdot v_3,\ \ldots,\ v_{10} = A \cdot v_9$

What numbers appear in the sequence of vectors $v_1, v_2, v_3, \ldots, v_{10}$?

Question 7

Consider a river of width 10m accross.
Suppose the depth of the river is given by $d(x) = x \cdot (10 - x)/25$ meters for $0 \le x \le 10$ and the velocity at position *x* is given by $v(x) = x \cdot (10 - x)/100$ m/s for $0 \le x \le 10.$.

Calculate the flow of the river exactly using an integral.
For $n = 3$ measurements taken at equally spaced intervals (at 2.5m, 5.0m, and 7.5m) use the Trapezoidal approximation to estimate the flow of the river and also the Midpoint approximation to estimate the flow.
Repeat this for $n = 7$ measurements.

Which approximation is better?

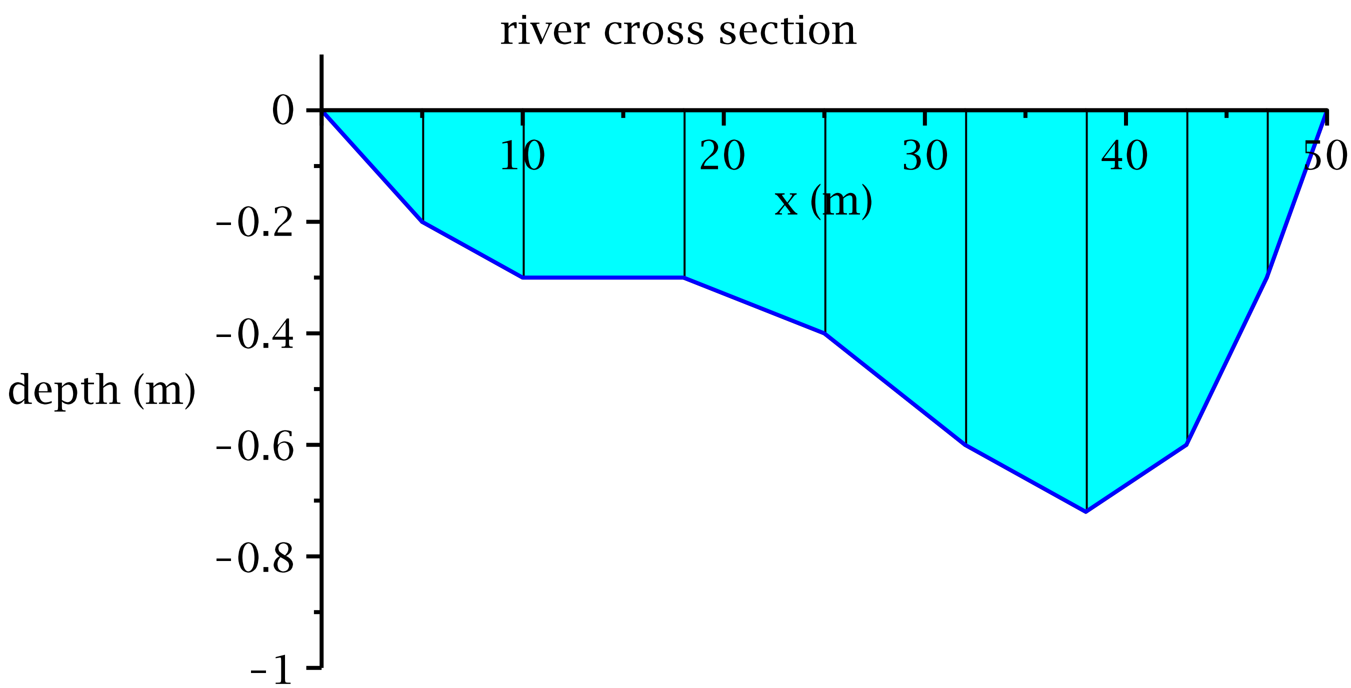
Question 8

Below is data for the velocity of a river that represents measurements taken at different positions accross the river. The data value $[x, y, v]$ means at position *x* meters the river is *y* meters deep and we measured the velocity of the water at position *x* at depth 40% above the river bed to be *v* meter per second.

```
> Data :=
  [ [0,0.0,0.0],
    [5,0.2,0.1],
    [10,0.25,0.2],
    [18,0.3,0.3],
    [25,0.4,0.4],
    [32,0.6,0.45],
    [38,0.72,0.50],
    [43,0.6,0.40],
    [47,0.3,0.30],
    [50,0.0,0.0] ];
```

$Data := [[0, 0., 0.], [5, 0.2, 0.1], [10, 0.25, 0.2], [18, 0.3, 0.3], [25, 0.4, 0.4], [32, 0.6, 0.45], [38, 0.72, 0.50], [43, 0.6, 0.40], [47, 0.3, 0.30], [50, 0., 0.]]$

Generate a plot of the cross section of the river. For each section of the river i.e. for $xL \le x \le xR$ with depths dL and dR generate a blue trapezoid and assemble the plot so that it looks like the figure below. See the **polygon** command in the **plottools** package.



Question 9

Consider a random walk in the XY plane where at each time step you walk one step (one unit) either to the left, right, up or down, at random. Starting from the origin, generate plots for two random walks with at least n=1000 random steps (n=10,000 is much better).

So first create a list of n values P := $[[0, 0], [x_1, y_1], [x_2, y_2], ..., [x_n, y_n]]$.
You can also use an array of points here here instead of a Maple list.
Then you can simply graph them using the `plot( P, style=line )`; command.
To get random numbers from 1,2,3,4 use the following

```
> R := rand(1..4):
```

Now when you call R() you will get one of 1,2,3,4 at random, e.g.,

```
> R(), R(), R();
```

3, 3, 2