Assignment 1, MACM 204, Fall 2017

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The purpose of this assignment is to learn the basics of Maple: how to input formulas, how to graph functions, solve equations, calculate derivatives and integrals, and to program some loops.

Due Tuesday September 19th at 3:00pm in the drop off box.
Do all calculations in Maple. Please print your Maple worksheets. You may compress the plots to save paper.

Question 1

Consider the function $f(x) = x\,\mathrm{e}^{-x}$. Construct a plot of [$f(x)$, $f\,'(x)$, $f\,''(x)$] on the same graph for the domain $0 \le x \le 10$.
Read the help page for `?plot,options`. Using the appropriate options, create a **legend** for the plot, and add a **title**.

Question 2

Using the **evalf** command, calculate the following values **to 20 decimal places.**

$\pi = 3.141\ldots$, $\mathrm{e}^2 = 7.389\ldots$, $\sqrt{2}$=1.414..., $\ln(2)$=0.693..., and $\int_0^1 \sin(x^2)\,\mathrm{d}x = 0.310\ldots$

Question 3

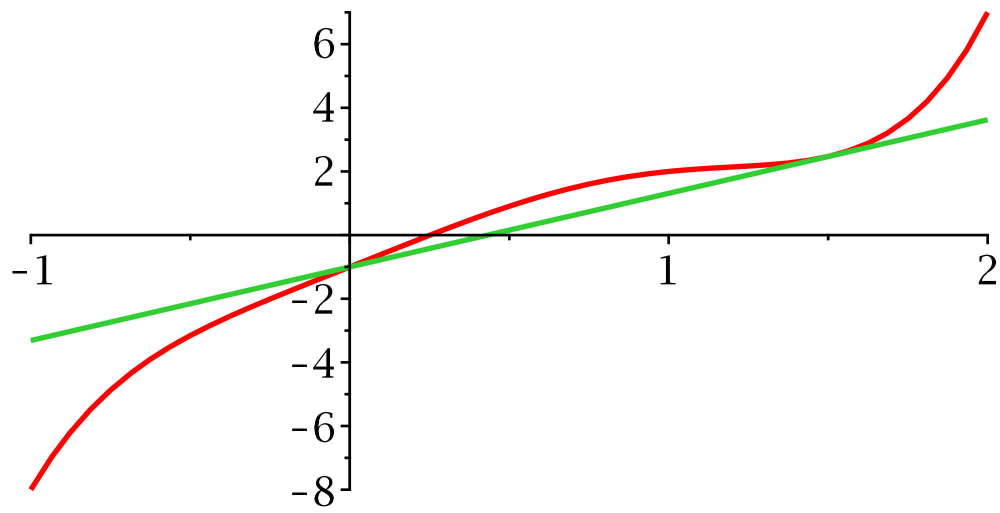
Consider the polynomials $f := x^4 - 1$ and $g := x^4 - 4\,x^3 + 8\,x - 4$ and
$h := x^4 - 4 \cdot x^3 + 4 \cdot x^2 - 8 \cdot x + 4$.
Graph each separately on a suitable domain for x so that we can see all real roots.
Now factor the polynomials using the **factor** command.
Why does the **factor** command not factor them into linear factors?
Now, using **solve**, solve for the roots. Using Maple, verify that the 4 roots of the polynomial g are correct.
Now, using **fsolve**, solve for numerical approximations of the roots, including the complex roots.
Read the help page for **?fsolve** to find out how to get the complex roots.

Question 4

Shown in the plot below is a plot of $f := x^5 - 2\,x^4 + 4\,x - 1$ on the domain -1 < x < 2.
Shown also is the line tangent to $f(x)$ at $x = \frac{3}{2}$.
Use Maple to reproduce the plot. Do this by using the diff and eval commands to compute the slope of $f(x)$ at $x = \frac{3}{2}$.
Now express the area enclosed by the tangent line and $f(x)$ as a definite integral.
Evaluate the definite integral using Maple.

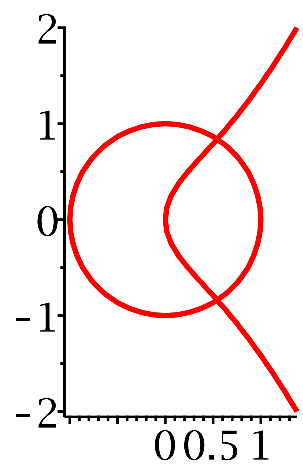
```
> f := x^5-2*x^4+4*x-1;
```

$f := x^5 - 2x^4 + 4x - 1$



Question 5

Consider the two equations $y^2 = x^3 + x$ and $x^2 + y^2 - 1$ which are the equations of an eliptic curve and a circle. Graph the two curves together using the **implicitplot** command in the **plots** package. Improve the smoothness of the plot using the **grid** option. The circle will appear distorted. Find out from ?plot,options how to scale the axes so that the circle looks like a circle (so that the plot looks like the figure below).



You can read off from the plot roughly where the curves intersect. Using the **fsolve** command compute the intersection points accurately.

Question 6

Calculate the following antiderivatives in Maple.

(i) $\int x^2 e^{-x} \, dx$ (ii) $\int x^2 \cos(x) \, dx$ (iii) $\int \sqrt{x} \cos(x) \, dx$.

To check that the results are correct, use Maple to differentiate each antiderivative.
Now try to find an antiderivative that Maple cannot do.

The last four questions are to get you to write some loops and to experiment.

Question 7

Consider the definite integral $\int_0^\infty x^n e^{-x}\,dx$.

Calculate the integral in Maple for n = 1, 2, 3, 4, 5 .
From these values deduce the formula for the integral as a function of n.

Question 8

The first n terms of the Taylor series for e^x are $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + ... + \frac{x^n}{n!}$.
Write a Maple **for loop** that constructs the Taylor polynomial for any value of n.
Don't use a Maple command that computes the Taylor polynomial, instead, construct it one term at a time by adding up the terms in a for loop. Run your code for $n = 10$.

Question 9

Consider the odd primes 3, 5, 7, 11, 13, 17, 19, etc.
Let S be the odd primes congruent to 1 mod 4 and T be the odd primes congruent to 3 mod 4.
So S = 5, 13, 17, 29, etc. and T = 3, 7, 11, 19, 23, etc.

Suppose we go through the odd primes in order counting the number of primes in S and T.
So at the first step S = { } and T = {3} so T has more primes than S.
At the second step S = {5} and T = {3} so S and T have the same number of primes.
At the third step S = {5} and T = {3,7} so T has more primes than S again.
Here is a little table counting the size of S and T for the first 10 primes

Prime	3	5	7	11	13	17	19	23	29	31
|S|	0	1	1	1	2	3	3	3	4	4
|T|	1	1	2	3	3	3	4	5	5	6

Notice that $|T| \geq |S|$ for the first 10 primes. Does it ever happen that $|S| \geq |T|$?
Using the builtin **nextprime** command write a loop that counts $|S|$ and $|T|$ for primes up to 10^6 and prints out the primes where $|S|$ **becomes** greater than $|T|$.
What do you conclude from the data you get?

Question 10

Let $A = \int_a^b f(x)\,dx$. Recall that the value of A may be approximated by the **Midpoint rule** on n intervals of width $h = \frac{(b-a)}{n}$ using the formula

$$M_n = h \cdot \left(f\left(a + \frac{h}{2}\right) + f\left(a + \frac{3}{2} \cdot h\right) + f\left(a + \frac{5}{2} \cdot h\right) + \ldots + f\left(a + \frac{2 \cdot n - 1}{2} \cdot h\right) \right).$$

For $f(x) = x^2 \cdot \sin(2 \cdot x)$ and $a = 0$, $b = 1$, calculate A using Maple to 10 decimal places by using Maple's integration command. Now using a loop, calculate M_8, M_{16} and M_{32} using 10 digit arithmetic (the default) .