# Assignment 1, MACM 204, Fall 2026

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The purpose of this assignment is to learn the basics of Maple: how to input formulas, graph functions, solve equations, calculate derivatives and integrals, and to program some loops.

Due 11pm Wednesday September 23rd. Late penalty: -10% per hour late.

Do all calculations in Maple. Please put your name and student number on top of each Maple worksheet, export your Maple worksheets to PDF files (on the MAC, print them to a PDF file) and upload them to Crowdmark to the corresponding question box.

## Question 1

Input the function $f(x) = x^2\, e^{-x}$ in Maple and calculate $f'(x)$ and $f''(x)$.
Graph $f(x)$ **and** $f'(x)$ on the same graph for the domain $0 \le x \le 8$.
Using an option, colour the two functions red and blue and include a suitable title.
Read the help page for `?plot,options`.
Now using Calculus and Maple, calculate the local maximum of $f(x)$.

## Solution 1

## Question 2

Using the **evalf** command, calculate the following values **to 20 significant digits.**

$\pi = 3.141\ldots$, $\sqrt{3} = 1.732\ldots$, $e = 2.718\ldots$, $\int_0^1 f(x)\,dx = 0.746\ldots$ where $f(x) = x^2 e^{-x}$ and $g''(1) = 10.66\ldots$ where $g(x) = \tan(x)$.

## Solution 2

## Question 3

Consider the polynomials $f = x^4 - 1$, $g = x^4 - 4x^3 + 8x - 4$ and $h = x^4 - 4 \cdot x^3 + 4 \cdot x^2 - 8 \cdot x + 4$.
Graph each separately on a suitable domain for x so that we can see all real roots.
Now, using **solve**, solve for the roots. Using Maple, verify that the 4 roots of the polynomial g are correct.
Now, using **fsolve**, solve for numerical approximations of the roots, including the complex roots.
Read the help page for **?fsolve** to find out how to get the complex roots.

## Solution 3

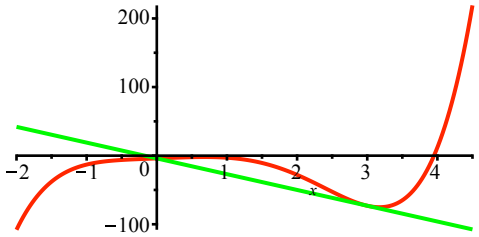
## Question 4

Shown in the plot below is a plot of $f := x^5 - 4x^4 + 4x - 4$ on the domain $-1 \le x \le 4.5$. Shown also is the line tangent to $f(x)$ at $x = 3$.

Use Maple to calculate $f'(3)$ so you can fine the tangent line. Then recreate the plot.

```
> f := x^5-4*x^4+4*x-4;
```

$$f := x^5 - 4x^4 + 4x - 4 \qquad (7.1)$$




## Solution 4

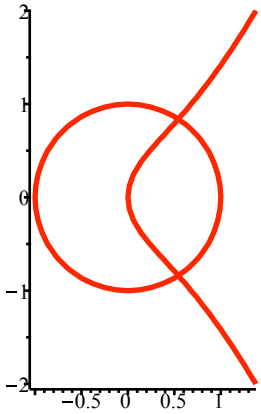
## Question 5

A car is travelling at velocity $v(t) = 30 \cdot t \cdot (4 - t)$ *kmph*.
What is the maximum velocity on $0 \le t \le 4$ ?
How far does the car travel on $0 \le t \le 4$ ?
Use Maple to do the calculations.

## Solution 5

## Question 6

Consider the two equations $y^2 = x^3 + x$ and $x^2 + y^2 = 1$ which are the equations of an eliptic curve and a circle. Graph the two curves together using the **implicitplot** command in the **plots** package. The circle will appear distorted. Find out from ?plot, options how to scale the axes so that the circle looks like a circle.




You can read off from the plot roughly where the curves intersect.
Use Maple to calculate the intersection points accurately.
Do this by solving two simultaneous equations using the **fsolve** command.

## Solution 6

The last four questions are to get you to write some loops and to experiment.

# Question 7

Suppose you are given a Maple list R of numbers which are the roots of a polynomial $f(x)$. Write a Maple for loop that constructs the polynomial. For example, if R = [1, 3, 5, 11, 12] then the value of $f$ should be the polynomial $(x-1)\,(x-3)\,(x-5)\,(x-11)\,(x-12)$.

Now suppose you are given a Maple list C of numbers which are the coefficients of a polynomial $g(x)$. Write a Maple for loop that contructs the polynomials. For example, if C = [1,4,0,5,2] then the value of $g$ should be $1 + 4\cdot x + 5\cdot x^3 + 2\cdot x^4$.

# Solution 7

# Question 8

Theorem: for all positive integers $m$ and $n$, $\int_{-\pi}^{\pi} \sin(m\cdot x)\cdot\cos(n\cdot x)\,dx = 0$ .

**Part (a)** I want you to learn to use Maple to check mathematical statements like this because it's so easy.
Using a double for loop, show that the theorem is true for all $1 \le n \le 5$, $1 \le m \le 5$.
Note, to force Maple to print something use the **print(**...) command.

**Part (b)** Now, having checked the theorem using Maple we really don't know why the theorem is true.
To see why the theorem is true, graph $\sin(m\cdot x)\cdot\cos(n\cdot x)$ for two choices of $m$ and $n$ .
Now in one or two sentences, explain why the integrals are zero.

# Solution 8

# Question 9

This is your first mathematical experiment. It's called the prime number race.
Consider the primes 2, 3, 5, 7, 11, 13, 17, 19, etc.
Let $P_i$ be the set of the first *i* primes. So $P_4$ = {2, 3, 5, 7}.
Let $S_i$ be the number of primes in $P_i$ which are congruent to 1 mod 4.
Let $T_i$ be the number of primes in $P_i$ which are congruents to 3 mod 4.
So $S_4 = 1$ since 5 mod 4 = 1 and $T_4 = 2$ since 3 mod 4 = 3 and 7 mod 4 = 3.

Suppose we go through the primes in order counting the number of primes $S_i$ and $T_i$.
I get the following data

| *i*, prime | 1, 2 | 2, 3 | 3, 5 | 4, 7 | 5, 11 | 6, 13 | 7, 17 | 8, 19 | 9, 23 | 10, 29 |
|---|---|---|---|---|---|---|---|---|---|---|
| $S_i$ | 0 | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 3 | 4 |
| $T_i$ | 0 | 1 | 1 | 2 | 3 | 3 | 3 | 4 | 5 | 6 |

Notice that $T_i \geq S_i$ for first 10 primes.
If you keep going you will notice that $T_i$ gets larger than $S_i$. I get $S_{1000} = 495$ and $T_{1000} = 505$.
Question: Does it ever happen that $S_i > T_i$ ? If so when? How often?

Write some Maple code that computes $S_i$ **and** $T_i$ and stops the first time $S_i > T_i$.
Print out at which prime this happens. Use the builtin **nextprime** command to get the primes.

Now write a second loop that goes through all primes up to 10^6 and calculates how often $S_i > T_i$.

# ▶ Solution 9

# ▼ Question 10

Let $A = \int_a^b f(x)\,dx$. Recall that the value of A may be approximated by the **Midpoint rule** on $n$ intervals of width $h = \frac{(b-a)}{n}$ using the formula

$$M_n = h \cdot \left( f\left(a + \frac{h}{2}\right) + f\left(a + h + \frac{h}{2}\right) + \ldots + f\left(a + (n-1)\cdot h + \frac{h}{2}\right) \right).$$

For $f(x) = x^2 \cdot \sin(2 \cdot x)$ and $a = 0$, $b = 1$, calculate $A$ using Maple to 10 decimal places by using Maple's integration command. Now using a Maple loop, calculate $M_8$, $M_{16}$ and $M_{32}$ using 10 digit arithmetic (the default) . **Note**, since $M_n$ is a numerical approximation to an area we want to do the computation using decimal arithmetic, not exact arithmetic.

# ▶ Solution 10