Assignment 3 MACM 204 Fall 2026

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Due 11pm Wednesday October 21st. Late penalty: -10% per hour late.

Answer all questions using Maple. Please do each question in a seperate worksheet.
Export them to .pdf files and upload to crowdmark.

Question 1

Suppose we toss a coin 10 times. Then we would expect to get 5 heads and 5 tails. But often we won't. Let's estimate the probability of getting 5 heads and 5 tails by doing a simple experiment. Toss a coin 10 times and count the number of heads h that we get. Repeat this 10,000 times in a loop and record how many times you get 5 heads and 5 tails. Estimate the probability of getting 5 heads and 5 tails when you toss a coin 10 times.

To simulate tossing a coin use a random bit generator such that a 1 is a head and a 0 is a tail. For example

```
> bit := rand(0..1):
> bit();
```

$$0 \tag{1.1}$$

```
> bit();
```

$$0 \tag{1.2}$$

```
> seq( bit(), i=1..10 );
```

$$0, 1, 0, 1, 1, 1, 0, 1, 1, 1 \tag{1.3}$$

I got 7 one bits so 7 heads. Let's try again

```
> seq( bit(), i=1..10 );
```

$$0, 0, 1, 0, 1, 0, 1, 0, 1, 1 \tag{1.4}$$

I got 75one bits so 5 heads. Let's try again

```
> seq( bit(), i=1..10 );
```

$$1, 1, 0, 0, 0, 1, 0, 1, 0, 1 \tag{1.5}$$

Write a Maple procedure Run(n) that repeats this experiment n times in a loop and outputs how many times you get 5 one bits (5 heads and 5 tails).
Execute your Maple procedure on the following

```
> Run(10000);
> Run(10000);
> Run(10000);
```

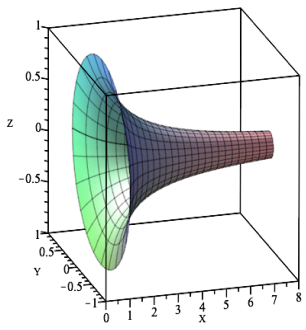
You should find that the probability is just under 1/4. If you've taken MACM 201 or STAT 270 you should be able to determine the exact probability.

Solution 1

Question 2

Part (a) Consider the parametric curve defined by $x = 5 \cdot \cos(\theta)$, $y = 3 \cdot \sin(\theta)$ for $0 \le \theta \le 2 \cdot \pi$. Graph the parametric curve using the **plot** command. What object is it?

Use the **scaling=constrained** option to get a 1 to 1 scaling so that you are not misled by the plot.
Part (b) Using the **spacecurve** command in the plots package, graph a circle of radius 1 in the YZ plane.
Use the labels option to put axes labels on the axes.
Part (c) The command **plot3d**([x(s,t),y(s,t),z(s,t)], s=a..b, t=c..d) creates a surface where the x, y, z co-ordinates are functions of two parameter s, t. On Assignment 2 you computed the volume of Gabriel's horn which is the solid created by rotating $1/x$ about the x axis for $1 \leq x \leq 8$. Here is the surface. Recreate the plot.



Solution 2

Question 3

Consider a river of width 10m accross.
Suppose the depth of the river is given by $d(x) = x \cdot (10 - x)/25$ meters for $0 \leq x \leq 10$
and the velocity at position x is given by $v(x) = x \cdot (10 - x)/100$ m/s for $0 \leq x \leq 10$. .

Part (a) Calculate the flow of the river exactly using an integral.
Part (b) For 3 measurements taken at equally spaced intervals (at 2.5m, 5.0m, and 7.5m) use the Trapezoidal approximation to estimate the flow of the river and also the Midpoint approximation to estimate the flow. See my lecture notes. **Repeat this** for 7 measurements taken at equally spaced intervals (1.25m appart).
You should find that the Midpoint approximation is significantly better.

Solution 3

Question 4

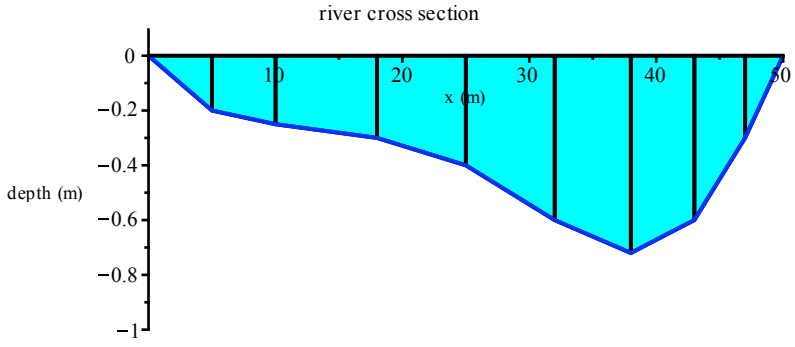
Below is data for the velocity of a river that represents measurements taken at different positions accross the river. The data value $[x, y, v]$ means at position x meters the river is y meters deep and we measured the velocity of the water at position x at depth 40% above the river bed to be v meters per second.

```
> Data :=
  [ [0,0.0,0.0],
    [5,0.2,0.1],
    [10,0.25,0.2],
```

```
    [18,0.3,0.3],
    [25,0.4,0.4],
    [32,0.6,0.45],
    [38,0.72,0.50],
    [43,0.6,0.45],
    [47,0.3,0.30],
    [50,0.0,0.0] ];
```

$$Data := [[0, 0., 0.], [5, 0.2, 0.1], [10, 0.25, 0.2], [18, 0.3, 0.3], [25, 0.4, 0.4], [32, 0.6, 0.45], [38, 0.72, 0.50], [43, 0.6, 0.45], [47, 0.3, 0.30], [50, 0., 0.]] \quad \textbf{(7.1)}$$

Generate a plot of the cross section of the river. For each section of the river i.e. for $xL \le x \le xR$ with depths dL and dR generate a blue trapezoid and assemble the plot so that it looks like the figure below. See the **polygon** command in the **plottools** package.



Solution 4

Question 5

Part (a)

Solve the following linear systems in Maple using the solve command.
For each system, how many solutions are there? (you have to interpret the output from Maple).

```
> sys1 := [x+y+z=2, x-y=1, x+z=0];
```

$$sys1 := [x + y + z = 2, x - y = 1, x + z = 0] \quad \textbf{(9.1)}$$

```
> sys2 := [x+y+z=2, x-y-2*z=1, 2*x-z=3];
```

$$sys2 := [x + y + z = 2, x - y - 2\,z = 1, 2\,x - z = 3] \quad \textbf{(9.2)}$$

```
> sys3 := [x+y+z=2, x-y=1, 2*y+z=-1];
```

$$sys3 := [x + y + z = 2, x - y = 1, 2\,y + z = -1] \quad \textbf{(9.3)}$$

Part (b)

Recall that an equation of the form $a \cdot x + b \cdot y + c \cdot z = d$ is the equation of a plane in 3 dimensions. Graph each system (of three planes) on the same plot using the **implicitplot3d** command in the plots package for a suitable domain. So you will have three plots. Give the three planes different colours. Rotate each plot so that we can see the solution(s) from solve, that is, where the planes intersect.

Solution 5

Question 6

Consider the 3 by 3 system of polynomial equations

$$\{x^2+y^2+z^2=4, x^2+y^2-z^2=4, x+y+z=0\}$$

Part (a). Solve the system using the **solve** command. You will get solutions involving RootOf($_Z^2-2$) which stands for $\pm\sqrt{2}$. To get solutions in terms of $\sqrt{2}$ use this option before you call solve.

```
> _EnvExplicit := true;
```

Part (b). Solve the system with the **fsolve** command. You may not get any solutions. Find out how to give fsolve an initial approximation of the solutions so that it will converge to the solutions.

Part (c). Using the **implicitplot3d** command in the plots package, generate a 3d plot of the three surfaces which include the real solutions to try to visualize the solutions.

Solution 6

Question 7

Consider the function

```
> f := 4*y^3+x^2-12*y^2-36*y+x^3+2;
```

$$f := x^3+4y^3+x^2-12y^2-36y+2 \qquad \textbf{(13.1)}$$

Find the critical points using Maple. You should get 4 of them. Determine which are saddle points, which are local minimums and which are local maximums.

Graph the surface $f(x,y)$ and highlight the critical points by drawing a spike (vertical line) through them using the **spacecurve** command in the plots package. Use blue for the saddles and red for the others so we can tell which is which.

Solution 7

Question 8

Consider the function

```
> f := 4*y^3+x^2-12*y^2-36*y+x^3+2;
```

$$f := x^3+4y^3+x^2-12y^2-36y+2 \qquad \textbf{(15.1)}$$

Part (a)

Construct the second degree Taylor polynomial $T2(x,y)$ for $f(x,y)$ about $x=0, y=0$.
The formula for T2 is in my notes on Canvas from the lecture on partial derivatives.
Graph the error $E(x,y)=f(x,y)-T2(x,y)$ on $-1\le x\le 1, -1\le y\le 1$.

Part (b)

You are to calculate the largest error of $E(x,y)$ on the unit disk $x^2+y^2\le 1$. Assuming the largest error is on the boundary of the disk, that is, on the unit circle $x^2+y^2=1$ we will use the parameterization of the unit circle, namely, $x=\cos(t), y=\sin(t)$ for $0\le t\le 2\cdot\pi$. Let $e(t)=E(\cos(t),\sin(t))$. To visualize the error graph

$[x=\cos(t), y=\sin(t), z=e(t)]$ for $0\le t\le 2\cdot\pi$

using the **spacecurve** command in the plots package together with a plot of $E(x,y)$ from Part (a).

Part (c)

Now calculate the maximum error for the function $e(t)$.
Just use Calculus for a function of one variable.

► Solution 8