Assignment 4, MACM 204, Fall 2026

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Please answer each question in a separate worksheet so you can upload each solution to Crowdmark.

Due 11pm Wednesday November 4th. Late penalty is -10% per day late.

```
> plots[setoptions](size=[450,300]);
```

Question 1

If you ask Maple to compute $\int \sin(x)\cdot\cos(x)\,dx$, Maple outputs $\frac{\sin(x)^2}{2}$ but if you ask Wolfram Alpha to compute the same integral, it outputs $-\frac{1}{2}\cdot\cos(x)^2$. You can get Maple to compute a third answer as follows.

```
> f := sin(x)*cos(x);
```

$$f := \sin(x)\cos(x)$$

```
> f := combine(f,trig);
```

$$f := \frac{\sin(2x)}{2}$$

```
> int(f,x);
```

$$-\frac{\cos(2x)}{4}$$

Show that all three antiderivatives $\frac{\sin(x)^2}{2}$, $-\frac{\cos(x)^2}{2}$ and $-\frac{\cos(2\cdot x)}{4}$ are different functions. Then show that all three antiderivatives are correct, first, algebraically and second, visually.

Solution 1

Question 2

Consider a random walk in the XY plane where at each time step you walk one step (one unit) either to the left, right, up or down, at random. Starting from the origin, generate plots for two random walks with at least n=1000 random steps.

So first create an **Array** of points P := $[[0,0],[x_1,y_1],[x_2,y_2],\ldots,[x_n,y_n]]$ where each point is a list of two numbers. Do not use a list for P.
Then you can simply graph the walk using the plot(convert(P,list), style=line); command.
The plot command wants a list so you will need to convert the Array to a list.

To get random numbers from 1,2,3,4 use the following

```
> R := rand(1..4):
```

Now when you call R() you will get one of 1,2,3,4 at random, e.g.,

```
> R(), R(), R();
```

$$1, 3, 3$$

Solution 2

Question 3

Start with an m by n two dimensional array A initialized as follows.

```
> m,n := 5,8;
  A := Array(1..m,1..n):
  A[1,3] := 1;
  A[1,6] := 1;
  A;
```

$$m, n := 5, 8$$

$$A_{1,3} := 1$$

$$A_{1,6} := 1$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Define the neighbours of column 1 to be columns n and 2, the neighbours of column n are columns $n-1$ and 1.
The neighbours of column i for $2 \le i \le n-1$ are columns $i-1$ and $i+1$.

Write a Maple procedure UpdateRow(A,r,n) that updates the A[r,i] for $1 \le i \le n$ to be the sum of the values of the neighbours mod 2 from row $r-1$ in A.
For example,
A[2,1] := A[1,8]+A[1,2] mod 2 = 0+0 mod 2 = 0.
A[2,2] := A[1,1]+A[1,3] mod 2 = 0+1 mod 2 = 1 and
A[2,3] := A[1,2]+A[1,4] mod 2 = 0+0 mod 2 = 0.
Execute your procedure on the above Matrix. You should get

```
> UpdateRow(A,2,n);
```

$$1$$

```
> A;
```

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> UpdateRow(A,3,n);
  UpdateRow(A,4,n);
  UpdateRow(A,5,n);
  A;
```

$$0$$

$$1$$

$$0$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now do the following experiment.

```
> m,n := 64,128;
  A := Array(1..m,1..n):
  A[1,64] := 1;
  for r from 2 to m do UpdateRow(A,r,n); od:
```

$$m, n := 64, 128$$

$$A_{1,\,64} := 1$$

To visualize the 0's and 1's in A use

```
> plots[listdensityplot](A);
```

You should get a fractal. Do you recognize it? Find out what it is.

Solution 3

Question 4

Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and vectors $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Part (a) Input A, u and w into Maple and calculate $2 \cdot u + w$, A^3 and A^{-1}.

Part (b) Using commands in the **LinearAlgebra** package calculate the determinant of A and solve the linear systems $A \cdot x = u$ and $A \cdot x = w$ for x. Note, to load the package and see all the commands in the package do

```
> with(LinearAlgebra):
> ?LinearAlgebra
```

Part (c) Starting with the vector $v_1 = [1, 0]$ compute, in a for loop, the vectors

$v_2 = A \cdot v_1$, $v_3 = A \cdot v_2$, $v_4 = A \cdot v_3$, ..., $v_{10} = A \cdot v_9$

What numbers appear in the sequence of vectors $v_1, v_2, v_3, ..., v_{10}$?

Part (d) Let I be the 2 by 2 identity matrix.
Construct I and the 2 by 4 "bookkeeping matrix" $[A \mid I]$.
Apply the command in the LinearAlgebra package that reduces the matrix $[A|I]$ to reduced row Echelon form. You should get $[I|A^{-1}]$. Extract the inverse

Solution 4

Question 5

Input **and evaluate** the following five integrals **using Maple's 2 dimensional input**.
I.e reproduce the Maple input and output below. On Windows, use F5 to go into 2D math input mode.
Use the Expression pallette to input an integral and the functions and the Common Symbols pallette for π, ∞.

> $\int \sqrt{1-x^2}\, dx$

$$\frac{x\sqrt{-x^2+1}}{2} + \frac{\arcsin(x)}{2}$$

> $\int \frac{4}{(x^3 - 4\cdot x)}\, dx$

$$\frac{\ln(x+2)}{2} - \ln(x) + \frac{\ln(x-2)}{2}$$

> $\int_0^1 \ln(x)\, dx$

$$-1$$

> $\int_0^\infty x^2 \cdot e^{-x}\, dx$

$$2$$

> $\int_0^{\frac{\pi}{2}} \sin(x)\cdot\cos(x)\, dx$

$$\frac{1}{2}$$

Input and evaluate the following four definite sums **using Maple's 2 dimensional input.**
Use the Expression pallette to input a sum. Reproduce the input and output below.

> $f := \sum_{k=1}^{n} (k^2)$

$$f := \frac{(n+1)^3}{3} - \frac{(n+1)^2}{2} + \frac{n}{6} + \frac{1}{6}$$

> **factor(f);**

$$\frac{n\,(n+1)\,(2\,n+1)}{6}$$

> $\sum_{k=0}^{n} \left(\binom{n}{k} \right)$

$$2^n$$

> $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} \right)$

$$\frac{\pi^2}{6}$$

```
> sum(1/n!, n=0..infinity);
```

$$\sum_{n=0}^{\infty} \left(\frac{1}{n!} \right)$$

$$e$$

Question 6

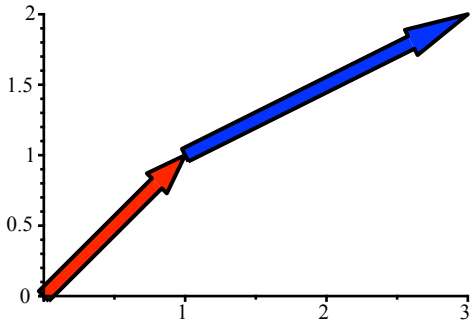
The following shows how to create a plot of two vectors

```
> u := <1,1>;
  v := <2,1>;
```

$$u := \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$v := \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

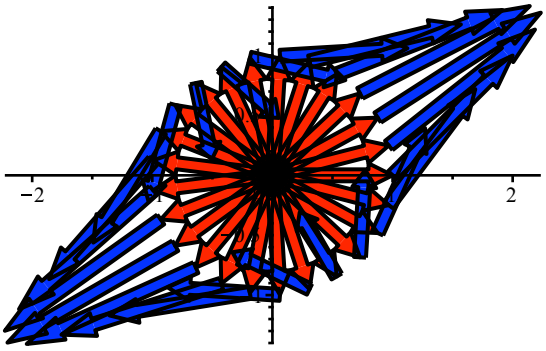
```
> with(plots):
> a1 := arrow( u, color=red, width=0.1 ):
> a2 := arrow( u,v, color=blue, width=0.1 ):
> display( [a1,a2], scaling=constrained );
```



Part (a). Using the trigonometric parameterization of the unit circle $[\cos(t), \sin(t)]$ for $0 \le t \le 2 \cdot \pi$ create $n = 21$ unit vectors as arrows equally spaced around the unit circle (see the red arrows in the figure below.)

Then multiply each unit vector u by the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ to get $v = A \cdot u$ and graph u together with v (in blue) to visualize the linear transformation $A \cdot u$ as shown in the figure below. **Now, from the figure, estimate the eigenvectors and eigenvalues.**

Part (b) Create an animation for the linear transformation where each frame in the animation is a pair of vectors $(u, A \cdot u)$. The animation parameter is t for $n = 100$ equally spaced values on $[0, 2 \cdot \pi]$.



Solution 6

Question 7

The purpose of this exercise is to introduce some new Maple commands for manipulating formulas. In the Maple code below I've used Maple to compute the partial fraction decomposition

$$f(x) = \frac{x+1}{x^2-4} = \frac{3}{4\,(x-2)} + \frac{1}{4\,(x+2)}$$

My code uses the numer, denom, factor, simplify, coeff, lhs, rhs, solve and subs commands.

```
> f := (x+1)/(x^2-4);
```

$$f := \frac{x+1}{x^2-4}$$

```
> n := numer(f);
```

$$n := x+1$$

```
> d := denom(f);
```

$$d := x^2-4$$

```
> factor(d);
```

$$(x-2)\,(x+2)$$

```
> Answer := A/(x-2)+B/(x+2);
```

$$Answer := \frac{A}{x-2} + \frac{B}{x+2}$$

```
> f = Answer;
```

$$\frac{x+1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

```
> eqn := simplify( d*(f=Answer) ); # clear fractions
```

$$eqn := x+1 = A\,(x+2) + B\,(x-2)$$

The next step is to equate coefficients in powers of x on both sides of the equation.

```
> lhs(eqn);
```

```
rhs(eqn);
```

$$A\,(x+2)+B\,(x-2)$$

```
> eqns := { coeff(lhs(eqn),x,1) = coeff(rhs(eqn),x,1),
            coeff(lhs(eqn),x,0) = coeff(rhs(eqn),x,0) };
```

$$eqns := \{1 = A + B,\ 1 = 2\,A - 2\,B\}$$

```
> sols := solve(eqns,{A,B});
```

$$sols := \left\{A = \frac{3}{4},\ B = \frac{1}{4}\right\}$$

```
> subs(sols,Answer); # you could also use eval(Answer,sol);
```

$$\frac{3}{4\,(x-2)} + \frac{1}{4\,(x+2)}$$

Now you do this for these rational functions.

```
> g := 4/(x^3-4*x);
```

$$g := \frac{4}{x^3 - 4\,x}$$

```
> h := 2/(x^3-4*x^2);
```

$$h := \frac{2}{x^3 - 4\,x^2}$$

▶ Solution 7