Assignment 5, MACM 204, Fall 2026

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Please answer each question in a separate worksheet so you can upload each solution to Crowdmark.

Due 11pm Wednesday November 18th. Late penalty: -10% per day late.

Question 1

Below are two Maple procedures f and g. Explain, in one sentence, or using a formula, what they compute.

```
> f := proc(L::list) local i,j;
      if L=[] then return 0 fi;
      j := 1;
      for i from 2 to nops(L) do
          if L[i]>L[j] then j := i; fi;
      od;
      j;
  end:
> g := proc(x::posint) local j,k;
      k := 0; j := 1;
      while j<x do k := k+1; j := 2*j; od;
      k;
  end:
```

Solution 1

Question 2

These are some questions to try out Maple's GraphTheory package.
To load the package and the SpecialGraphs subpackage.

```
> with(GraphTheory):
```

Part (a). Use the Graph command to create two graphs G1 and G2, both of which have vertices { 1, 2,3,4,5,6 } where
The set of edges of G1 and G2 are

```
> E1 := {{1,2},{2,3},{3,4},{4,5},{5,6},{6,1},{1,4},{2,5},{3,6}};
```

$$E1 := \{\{1,2\},\{1,4\},\{1,6\},\{2,3\},\{2,5\},\{3,4\},\{3,6\},\{4,5\},\{5,6\}\} \quad \textbf{(3.1)}$$

```
> E2 := {{1,2},{2,3},{3,4},{4,5},{5,6},{6,1},{1,3},{3,5},{5,1}};
```

$$E2 := \{\{1,2\},\{1,3\},\{1,5\},\{1,6\},\{2,3\},\{3,4\},\{3,5\},\{4,5\},\{5,6\}\} \quad \textbf{(3.2)}$$

respectively. Use the DrawGraph command to draw the graphs using the style=spring option.
Notice that both graphs have the same number of vertices and edges. Why are the two graphs regular? Why are they not the same?
Now test if G1 and G2 are planar using the IsPlanar command.
Use the IsVertexColorable command to test if G1 and G2 are 2 colorable, and if so, find a coloring. If the graph is two colorable use the HighlightVertex command to color vertices of one color red and the other color blue then redraw the graph so we can see the coloring.

Part (b). A graph with n vertices and $n-1$ edges which is connected is called a tree. There are 6

different trees with 6 vertices. Use the NonIsomorphicGraphs command to construct all 6 different trees with 6 vertices. Use the DrawGraph command to draw the 6 graphs that you get.

Part (c)

```
> with(SpecialGraphs);
> G := DodecahedronGraph();
```

$G :=$ *Graph 3: an undirected unweighted graph with 20 vertices and 30 edge(s)* **(3.3)**

Draw the dodecahedron graph with the DrawGraph command using (i) no options, (ii) the style= planar option, and (iii) the style=spring option.
The dodecahedron graph has Hamiltonian cycles (a walk through the vertices of G that goes through each vertex exactly once and returns to the starting vertex) and you might like to try to find one find.
The command

```
> IsHamiltonian(G,'C');
```

returns one Hamiltonian cycle in C. Use the HighlightEdges command to color the edges of the Hamiltonian cycle C red and then redraw the graph so we can see it.

Solution 2

Question 3

Write a Maple procedure Remove(A,x) that on input of an array A indexed from 1 and a value x, outputs a new array where all instances of x have been removed from A. For example

```
> A := Array(1..7,[1,2,3,2,3,2,1]);
```

$$A := \left[\begin{array}{ccccccc} 1 & 2 & 3 & 2 & 3 & 2 & 1 \end{array}\right] \quad \textbf{(5.1)}$$

```
> Remove(A,2);
```

$$\left[\begin{array}{cccc} 1 & 3 & 3 & 1 \end{array}\right] \quad \textbf{(5.2)}$$

Solution 3

Question 4

Consider the differential equation $y'(t) = 0.1 \cdot y \cdot (10 - y)$. Input the differential equation in Maple and solve it for the initial values $y(0) = 1$, $y(0) = 5$ and $y(0) = 10$. Graph the three solutions for $-2 \le t \le 8$ on the same graph. Include a legend in the plot.

Solution 4

Question 5

Let $M(t)$ be the amount owed on a 30 year mortgage of \$200,000 at time t years.
Suppose the annual interest rate on the mortgage is $r = 6\%$.
Suppose the term of the mortgage is 30 years, i.e., M(30) should be 0.
Suppose we pay \$$P$ per year.
We can model the change in what we owe the bank with the differential equation

$M'(t) = r \cdot M(t) - P$ dollars **per year**

and initial value $M(0)$ =$200000.
This model assumes the interest is charged continuously (banks usually charge interest daily which is approximately continuous) and if we assume we make the payments continuously (banks usually require us to pay monthly or weekly which is approximately continuous over 30 years). So the values we obtain with it will be approximations.
Part (a) input and solve the differential equation with $r = 6\%$ and $M(0) = 200000$.
Part (b) determine the anual payment P by solving $M(30) = 0$ for P.
Part (c) calculate the interest paid to the bank $30 \cdot P - 200000$
Suppose we want to pay down the mortgage faster to reduce the amount of interest.
Part (d) solve the differential equation with $P = 16000$ and graph $M(t)$ on a suitable domain
Part (e) determine when $M(t) = 0$ and calculate the interest paid to the bank.

Solution 5

Question 6

Suppose we have a 400 liter tank. Suppose 8 litres per minute of salt water (brine) flows into the tank at the top and then flows out of the tank at the bottom. Assume for simplicity that the salt water in the tank is stirred so that its concentration is uniform in the tank. Let $S(t)$ be the amount of salt, in grams, in the tank at time t minutes. Suppose the salt water flowing into the tank has concentration 100 grams per liter.

Find the differential equation to model the change in S(t).
Assuming there is no salt in the tank at time t=0 solve the differential equation using Maple.
What is S(∞)? That is, how much salt is in the tank after a long time?
Now graph S(t) for a suitable domain.

Solution 6

Question 7

The logistic growth with harvesting model for a population $y(t)$ at time t is given by
$y'(t) = \boldsymbol{a} \cdot y(t) \cdot \left(Y_{\max} - y(t) \right) - H$
Here *Ymax* is the maximum sustainable population of the environment, **a** is a constant and H is a constant harvesting rate. For Ymax = 8000, a=0.0001, and H = 1000, using the DEplot command, graph y(t) for $0 \le t \le 10$ for the initial values $y(0)$ in 1000, 4000, 8000 and 10000.

Now determine populations y for which $y' = 0$, i.e., find the initial polulations for which there is no growth or decline. You should get two. Graph these on the same graph - you should get two straight lines.

Solution 7

Question 8

```
> restart;
```

```
> for k to 5 do
      s := factor( sum( i^k, i=0..n ) );
      print( Sum(i^k,i=0..n) = s );
  od:
```

$$\sum_{i=0}^{n} i = \frac{n\,(n+1)}{2}$$

$$\sum_{i=0}^{n} i^2 = \frac{n\,(n+1)\,(2\,n+1)}{6}$$

$$\sum_{i=0}^{n} i^3 = \frac{(n+1)^2\,n^2}{4}$$

$$\sum_{i=0}^{n} i^4 = \frac{n\,(n+1)\,(2\,n+1)\,\left(3\,n^2+3\,n-1\right)}{30}$$

$$\sum_{i=0}^{n} i^5 = \frac{(n+1)^2\,n^2\,\left(2\,n^2+2\,n-1\right)}{12} \qquad \textbf{(15.1)}$$

In the above I've computed formulas $s(n)$ for $\sum_{i=0}^{n}\left(i^k\right)$. How can we check the formulas?

We can use mathematical induction to prove them all quite easily. We need to check that the formula is correct for $n=1$. We have $\sum_{i=0}^{1}\left(i^k\right)=1$ so we just check that $s(1)=1$. This is the base of the proof by induction. Now assume $\sum_{i=0}^{n}\left(i^k\right)=s(n)$. That's our induction hypothesis. For the induction step we need to show $\sum_{i=0}^{n+1}\left(i^k\right)=\sum_{i=0}^{n}\left(i^k\right)+(n+1)^k=s(n+1)$. Applying the induction hypothesis we just need to check if $s(n)+(n+1)^k=s(n+1)$. Do this. Test the formulas up to $k=10$.

▶ Solution 8