(i) Let R be a ring (field). R is a <u>differential ring (field</u>
if $\exists D : R \to R$ s.t. $\forall f, g \in R$

(i) $D(f+g) = D(f) + D(g)$ and

(ii) $D(f \cdot g) = D(f) \cdot g + f \cdot D(g)$.

(ii) Let F and G be differential fields with D_F and D_G.
G is a <u>differential extension</u> of F if

(i) $F \subset G$ and (ii) $\forall f \in F$ $D_F(f) = D_G(f)$.

(iii) Let $F(\theta)$ be a differential extension of F.
θ is <u>logarithmic</u> over F if $\exists u \in F$ s.t. $\theta' = \frac{u'}{u}$,
θ is <u>exponential</u> over F if $\exists u \in F$ s.t. $\theta' = u'\theta$,
θ is <u>algebraic</u> over F if $\exists p \in F[z]$ s.t. $p(\theta) = 0$.
θ is <u>transcendental</u> over F if θ is NOT algebraic.

(iv) $G = F(\theta_1, \theta_2, \dots, \theta_n)$ is an <u>elementary extension</u> of F if θ_i is logarithmic, exponential or algebraic over $F(\theta_1, \dots, \theta_{i-1})$ for $i = 1, 2, \dots, n$.

(v) The set of <u>elementary functions</u> of x is

$$E = \{ f : f \in \mathbb{C}(x)(\theta_1, \dots, \theta_n) = G \}$$ where G is an elementary extension of $\mathbb{C}(x)$.

E.g. $e^x \ln(1+x) \in \mathbb{Q}(x)(\theta_1 = e^x, \theta_2 = \ln(1+x)) \subset E$.
E.g. $\sin(x) = [e^{-ix} - ie^{-ix}]/2i \in \mathbb{Q}(i)(x)(e^{ix}) \subset E$

E.g. $e^{e^{\sqrt{x}}} \in \mathbb{Q}(x)(\theta_1 = \sqrt{x}, \theta_2 = e^{\sqrt{x}}, \theta_3 = e^{\theta_2}) \subset E$.