

Lec21C Handouts

March 30, 2021 1:48 PM

```

> restart;
> P := x^2;
> Q := x^5-3*x^4+x^3+5*x^2-6*x+2;
> sqrfree(Q);
      [1, [[x^2 - 2, 1], [x - 1, 3]]]

> T := (x^2-2); k := 3; qk := x-1;
      T := x^2 - 2
      k := 3
      qk := x - 1

> sigma*T*diff(qk,x) + tau*qk = P;
      sigma(x^2 - 2) + tau(x - 1) = x^2

> gcdex(T*diff(qk,x),qk,P,x,'sigma','tau');
> sigma;
      -1

> tau;
      2 + 2x

> INT := sigma/qk^(k-1)/(1-k) + Int( tau/T/qk^(k-1), x ):
> Int( P/factor(Q), x ) = INT;
      \int \frac{x^2}{(x^2 - 2)(x - 1)^3} dx = \frac{1}{2(x - 1)^2} + \int \frac{2 + 2x}{(x^2 - 2)(x - 1)^2} dx

> simplify( diff(INT,x)-P/Q );
      0

> P := tau;
> Q := qk^(k-1)*T;
      P := 2 + 2x
      Q := (x^2 - 2)(x - 1)^2

> T := (x^2-2); k := 2; qk := x-1;
      T := x^2 - 2
      k := 2
      qk := x - 1

> gcdex(T*diff(qk,x),qk,P,x,'sigma','tau');
> sigma;
      -4

> tau;
      6 + 4x

> INT := sigma/qk^(k-1)/(1-k) + Int( tau/T/qk^(k-1), x ):

```

$\text{gcdex}(A, B, x, 'S', 'T') \rightarrow G$
 $SA + TB = G$
 $\text{gcdex}(A, B, C, x, 'S', 'T')$
 solves
 $\int SA + TB = C$ for S, T .

Int(P/Q, x) = INT;

$$\int \frac{2+2x}{(x^2-2)(x-1)^2} dx = \frac{4}{x-1} + \int \frac{6+4x}{(x^2-2)(x-1)} dx$$

```
> restart;
> P := x^2;
> Q := (x-1)^3*(x^2-2);
```

$$P := x^2$$

$$Q := (x-1)^3 (x^2-2)$$

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> B := gcd(Q,diff(Q,x));
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$$B := (x-1)^2$$

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> alias(D=DD):
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> D := Q/B;
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$$D := (x-1)(x^2-2)$$

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> H := normal(D*diff(B,x)/B);
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$$H := 2x^2 - 4$$

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deg(A) < deg(B), deg(C) < deg(D)
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> A := a[0]+a[1]*x;
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$$A := a_0 + a_1 x$$

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> C := c[0]+c[1]*x+c[2]*x^2;
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$$C := c_0 + c_1 x + c_2 x^2$$

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> ANSATZ := A/B+Int(C/D,x);
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$$ANSATZ := \frac{a_0 + a_1 x}{(x-1)^2} + \int \frac{c_0 + c_1 x + c_2 x^2}{(x-1)(x^2-2)} dx$$

```
> P=diff(A,x)*D-A*H+C*B;
```

$$x^2 = a_1 (x-1)(x^2-2) - (a_0 + a_1 x)(2x^2-4) + (c_0 + c_1 x + c_2 x^2)(x-1)^2$$

```
> zero := collect(P-diff(A,x)*D+A*H-C*B,x);
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$$zero := -c_2 x^4 + (a_1 - c_1 + 2c_2)x^3 + (1 + a_1 + 2a_0 - c_0 + 2c_1 - c_2)x^2 + (-2a_1 + 2c_0 - c_1)x - 2a_1 - 4a_0 - c_0$$

```
> eqns := {coeffs(zero,x)};
```

$$eqns := \{-2a_1 + 2c_0 - c_1, a_1 - c_1 + 2c_2, -2a_1 - 4a_0 - c_0, 1 + a_1 + 2a_0 - c_0 + 2c_1 - c_2, -c_2\}$$

```
> sol := solve(eqns);
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$$sol := \left\{ c_2 = 0, a_0 = -\frac{7}{2}, c_1 = 4, c_0 = 6, a_1 = 4 \right\}$$

```
> ans := sort( eval(ANSATZ,sol), x );
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normal (f(x)) simplified
rational functions of x.

$$ans := \frac{4x - \frac{7}{2}}{(x-1)^2} + \int \frac{4x+6}{(x-1)(x^2-2)} dx$$

```
> simplify( diff(ans,x) - P/Q );
```

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