Theorem 12.2 (differentiation of logarithmic polynomials)

Let F be a differential field and $F(\theta)$ be a logarithmic transcendental differential extension of F with $\theta' \neq 0$.
I.e. $\theta = \log(u)$ for some $u \in F$, $u' \neq 0$, $\theta \notin F$.

If $a = a_n\theta^n + \cdots + a_1\theta + a_0 \in F[\theta]$ with $n > 0$, $a_n \neq 0$

(i) $a' = \frac{d\,a(\theta)}{dx} \in F[\theta]$
(ii) if $a_n' = 0$ then $\deg_\theta a' = n-1$
(iii) if $a_n' \neq 0$ then $\deg_\theta a' = n$

Theorem 12.3 (differentiation of exponential polynomials)

Let F be a differential field and $F(\theta)$ be an exponential transcendental differential extension of F with $\theta' \neq 0$.
I.e. $\theta = e^u$ for some $u \in F$, $u' \neq 0$, $\theta \notin F$

(i) If $a = a_n\theta^n + \cdots + a_1\theta + a_0 \in F[\theta]$ with $n > 0$, $a_n \neq 0$

$$a' = \frac{d\,a(\theta)}{dx} \in F[\theta] \text{ and } \deg_\theta a' = n$$

(ii) If $h \in F \setminus \{0\}$ and $m \in \mathbb{Z} \setminus \{0\}$ then

$$(h\theta^m)' = \bar{h}\theta^m \text{ for some } \bar{h} \in F \setminus \{0\}$$

(iii) $a(\theta) \mid a'(\theta) \Rightarrow a(\theta) = h\theta^m$ for some $h \in F$, $m \in \mathbb{Z}$