MATH 152 Assignment 8, Fall 2026.

The Final Exam is on Tuesday December 15th from 8:30am–11:30am.
About 20% of the final exam mark (8% of your course grade) will be based on the material covered in this assignment.

This assignment has computing exerises. It is worth 3% of your course grade.

For this Assignment, use **FormulaSheet4.pdf**. It's on Canvas.
FormulSheet4.pdf is the one I will put on the final exam.

Webassign Exercises

11.10 Exercises 55, 66, 86

11.11 Exercises 9, 13, 28

8.3 Exercises 22, 23, 25

Written Exercises

1 Section 11.10. Consider $f(x) = e^{-x}\cos x$.

(a) Use Wolfram Alpha to calculate the Taylor series for $f(x)$ at $x = 0$.
What are the Taylor polynomials $T_1(x), T_2(x), T_3(x), T_4(x)$ and $T_5(x)$?

(b) Use Wolfram Alpha to plot $f(x), T_1(x), T_3(x)$ and $T_5(x)$ together for $-2.5 \le x \le 2.5$. Upload a screenshot to Crowdmark. In one or two sentences, what can you say about the Taylor polynomials from the plot.

2 Section 11.10 Use multiplication of power series to find the first three non-zero terms in the Taylor series for $f(x) = e^{-x}\cos x$ and $g(x) = e^x \ln(1 + x)$ and at $x = 0$. See Example 15 of Section 11.10. Table 1 on page 804 has the Taylor series for $\ln(1 + x)$, e^x, and $\cos x$.

3 Section 11.10 Use series division to calculate the Taylor polynomial $T_4(x)$ for $f(x) = x/\sin x$ and $g(x) = \sec x = 1/\cos x$ about $x = 0$. Use Table 1 on page 804 for the Taylor series for $\sin x$ and $\cos x$. I suggest you check your answers using Wolfram Alpha. See my "Taylor Series" video on Canvas under Lecture Schedule and Material.

4 Section 11.11. $T_4(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$ is the degree 4 Taylor polynomial for $f(x) = e^x$. It can be used to approximate e^x for small x.

(a) Calculate $e^{0.4}$ and $T_4(0.4)$ and $e^{0.1}$ and $T_4(0.1)$. What are the actual errors?

(b) Use Taylor's inequality on page 762 to bound the error of $T_4(0.4)$ and $T_4(0.1)$.

Notice that the error bound for $T_4(0.1)$ is a lot less than for $T_4(0.4)$. To exploit this we use the identity $e^x = (e^{x/2})^2$ to approximate $e^{0.4}$ using

$$e^{0.4} = (e^{0.2})^2 = ((e^{0.1})^2)^2 \approx (T_4(0.1)^2)^2.$$

(c) Calculate $(T_4(0.1)^2)^2$. How many decimal places of accuracy do you get for $e^{0.4}$?

This is one of the ways a calculator computes e^x. It uses the identity $e^x = (e^{x/2})^2$ for large x and a Taylor polynomial $T_n(x)$ for small x.

5 Section 11.11

Recall that the sine integral $\mathrm{Si}(x)$ is defined by $\mathrm{Si}(x) = \int_0^x \frac{\sin(t)}{t} dt$. Let $T_n(x)$ be the degree n Taylor polynomial for $\mathrm{Si}(x)$ about $x = 0$.

(a) Calculate the Taylor series for $f(t) = \sin(t)/t$ about $t = 0$ up to the term of degree 6 and express the series in Sigma notation.

(b) The FTC part 1 says if $g(x) = \int_0^x f(t)dt$ then $g'(x) = f(x)$. So $\mathrm{Si}'(x) = \sin(x)/x$. To calculate $\mathrm{Si}'(0)$ we cannot use $\sin(0)/0$. We will use $\mathrm{Si}'(0) = \lim_{x\to 0} \sin(x)/x$. Calculate the limit using the Taylor series from part (a).

(c) Now integrate the series from part (a) to find the Taylor series for $\mathrm{Si}(x) = \int_0^x \sin(t)/t\, dt$ Give the terms up to degree 7 and express the series in Sigma notation.

(d) Calculate $T_5(1.0)$ to 7 decimal places to approximate $\mathrm{Si}(1.0)$.
Use Wolfram Alpha to compute $\mathrm{Si}(1.0)$ to 10 decimal places.

(e) The error in $T_5(x)$ is given by $|Si(x) - T_5(x)|$. Plot $Si(x) - T_5(x)$ in Wolfram Alpha on the interval $[-\pi, \pi]$. Upload a screenshot of your plot to Crowdmark. Where is the maximum error for $[-\pi, \pi]$?

(f) Bound the error of $T_5(1.0)$ by using the fact that the series for $\mathrm{Si}(x)$ is alternating. How many decimal places is $T_5(1.0)$ accurate to?

6 (Section 8.3) Sketch the region bounded by $y = \sqrt{x}, y = 0$ on $0 \le x \le 1$ and locate the centroid on your sketch. Now calculate the exact location of the centroid.

7 (Section 8.3) Sketch the region bounded by $y = \sin x, y = 0, x = 0, x = \pi$ and locate the centroid on your sketch. Now calculate the exact location of the centroid. You should get $\bar{x} = \pi/2$ and $\bar{y} = \pi/8$.