

MATH 340 Assignment 5, Fall 2010

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This assignment is due Wednesday November 3rd at 11:20am.

For problems involving Maple please submit a printout of a Maple worksheet.

Late penalty: -20% for up to 24 hours late. Zero for more than 24 hours late.

Section 2.3: Review of Vector Spaces

Exercises 2, 7, 9.

Also exercise 18 of section 2.1.

Section 2.6: Irreducible Polynomials

Exercises 1, 2, 6, 9, 10.

Do questions 1, 2 and 9 by hand. Use the `Factor` command in Maple to answer question 10.

Complex numbers

1. Let $i = \sqrt{-1}$, $a = (2 + 3i)$ and $b = (1 - 2i)$. Calculate $a + b$, ab and a/b .
2. Convert $a = 2 - 2i$ and $b = 2i$ to polar co-ordinates and calculate a^2 , ab and a/b in polar form. By hand, draw the points a, b, a^2, ab and a/b in the complex plane.
3. Let $z_1 = r(\cos \theta + i \sin \theta)$ and $z_2 = s(\cos \omega + i \sin \omega)$. Show that $z_1/z_2 = r/s [\cos(\theta - \omega) + i \sin(\theta - \omega)]$.
4. Let $\mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z} \text{ and } i^2 = -1\}$ and let addition and multiplication in $\mathbb{Z}[i]$ be defined as for \mathbb{C} . The set $\mathbb{Z}[i]$ is called the set of Gaussian integers. Show that $\mathbb{Z}[i]$ is a subring of \mathbb{C} . See Lemma 2.2.4.

The Fundamental Theorem of Algebra

1. By hand, calculate the complex eigenvalues AND eigenvectors of the matrix

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

2. Using Maple, factor the polynomials $x^4 - 3x^2 + 1$ and $x^4 + x^2 + 1$ over \mathbb{Q} . Then determine all real and complex roots by applying the quadratic formula by hand.
3. Determine all real and complex roots of the polynomials $x^4 + 3x^2 + 1$ and $x^5 - 1$ in Maple using both the `solve` and `fsolve` commands. Observe that $f(a + bi) = 0 \implies f(a - bi) = 0$.