Due Date: Tuesday April 6th, 2021, 11:00pm

Instructions

Answer all questions on paper or a tablet using your own handwriting. **Write your name**, student ID number **on the cover page** and list the questions you answered. If you use paper make a photo of each page and upload your solutions to crowdmark. If you use a tablet, export your assignment to .pdf and upload the .pdf to crowdmark.

Textbook Reading

- Sections: 11.4, 11.5, 12.1

Exercises

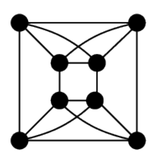
A. **Textbook Questions**

11.4 Exercises 14b, 19, 26abc.
11.5 Exercises 3e, 6.
12.1 Exercises 3, 10

B. **Instructor Questions**

Questions on 11.4

1. Let $P = (V, E)$ be the graph given by vertices $V = \{1, 2, 3, 4, 5\}$ and edges $E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 1\}, \{1, 5\}, \{2, 5\}, \{3, 5\}, \{4, 5\}\}$.
   (a) Draw a planar embedding of P with straight lines.
   (b) Draw a planar embedding of P^* the dual of P.
2. Find a subgraph of the graph G below that is subdivision of $K_{3,3}$. Conclude that G is not planar.

$G =$ 

3. Let $G = (V, E)$ be a connected planar simple graph with $|V| \geq 3$. In class we proved the bound $|E| \leq 3|V| - 6$. A bound is said to be "tight" if there is an example which equals the bound. Show that the bound $|E| \leq 3|V| - 6$ is tight, that is find a connected planar simple graph with $|V| = 6$ and $|E| = 3|V| - 6 = 12$ edges.

Questions on 11.5

4. Let $G = (V, E)$ a simple connected bipartite graph with V partitioned as $V = V_1 \cup V_2$. Prove or disprove the following
   (a) $|V_1| = |V_2|$ and $|V_1| \geq 2$ are necessary conditions for G to have a Hamiltonian cycle.
   (b) $|V_1| = |V_2|$ and $|V_1| \geq 2$ are sufficient conditions for G to have a Hamiltonian cycle.
5. State the converse of Theorem 11.8.
   Show that the converse of Theorem 11.8 is false by finding a counter example.
6. Twelve students are to be seated in a row of 12 seats in a classroom. If each student knows at most 5 others, explain why it is possible to seat them in such a way that each student does not know the student(s) sitting beside them.

Questions on 12.1

7. If a tree has three vertices of degree 2, four of degree 3, and two of degree 4, how many leaf vertices does it have?
8. Let $T = (V, E)$ be a tree. User Euler's theorem for planar graphs to show that $|V| = |E| + 1$.
9. Let T be a tree with at least 2 vertices. Prove that removing any edge disconnects T.