Due Date: January 25th, 2021, 11:00pm

Instructions

Answer all questions on paper or on a tablet using your own handwriting. Include a cover page with your name, student ID and a list of the questions you have answered.

If you use paper, take a photo of each page using your phone or scan each page and upload your work to crowdmark. If you use a tablet, export your work to a .pdf file and upload the file(s) to crowdmark.

Please number each page so that if you upload your pages in the wrong order we can find your answers.

You are expected to attempt all questions, including the questions from the textbook and the instructor's questions. We will only mark about 5 questions. We will post solutions to all questions on Canvas by 9am the day after the assignment is due. For this reason we cannot give extensions to assignments.

Don't forget to write **your name** and **student ID number** on the **cover page**!

Textbook Reading

- Sections: 1.1, 1.2, 1.3, 1.4

Exercises

A. **Textbook Questions : Grimaldi, 5th edition**

Section 1.2 Exercises 7, 10, and 15.
Section 1.3 Exercises 4 and 30.
Section 1.4 Exercises 2, 12, and 15.

B. **Instructors Questions**

1. List all binary strings of length 5 with three 1 bits. You should get $\binom{5}{3}$ of them.
2. There are n balls labelled $1, 2, 3, \ldots, n$ and n boxes labelled $1, 2, 3, \ldots, n$.
   (a) How many ways are there to put the balls into the boxes?
   (b) How many ways are there if each box must have one ball?
3. How many palindromes of length n are there for the alphabet $\Sigma = \{A, B, C, D\}$.
   A palindrome is a word that reads the same forwards as backwards, e.g. $ABBA$.
4. Consider the graph $G = (V, E)$ given by vertexes $V = \{1, 2, 3, 4, 5, 6\}$
   and edges $E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{5, 1\}, \{2, 5\}, \{1, 6\}\}$.
   Draw G and draw all paths of length 5 in G.
5. Draw a graph with 6 vertices where each vertex has degree 3. Draw a different graph with 6 vertices where each vertex has degree 3. Explain how they are different.
6. Draw all unlabelled trees with 5 vertices. You should get 3 trees.
7. Write out the proof that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ from the class notes.
8. Show that $\binom{n}{2} + \binom{n-1}{2}$ is a perfect square.
9. A box has 10 red balls, 10 green balls, 10 blue balls and 10 yellow balls. Each set of 10 balls is numbered 1 to 10. Suppose six balls are chosen from the box.
   (a) How many ways can there be six of the same colour?
   (b) How many ways can there be four of one colour and two of another colour?
   (c) How many ways can there be three of one colour and three of a different colour.
   (d) How many ways can there be two of one colour, two of a second colour, and two of a third colour?

10. We will count the number of cycles in the complete graph K_6.
    (a) How many cycles on 6 vertices are in K_6?
    (b) How many cycles on 5 vertices are in K_6?
    (c) How many cycles on 4 vertices are in K_6?
    (d) How many cycles on 3 vertices are in K_6?
11. Consider the expressions $f = (x + y + z)^7$ and $g = (x + y - 3z)^7$.
    (a) Find the coefficient of $x^2y^3z^2$ in f and g?
    (b) What is the sum of all the coefficients in f and g?
    (c) How many terms are there in the expansion of f and g?
    Note $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ has 4 terms $x^3, 3x^2y, 3xy^2$ and y^3.
12. (a) How many ways can Mary distribute 12 apples and 10 oranges to her three children?
    (b) How many ways can Mary distribute 12 apples and 10 oranges to her three children such that each child gets at least two apples and at least one orange.