A5S21 Text Solutions

October 22, 2020 1:41 PM

9.1 #2(a). $A(x) = (1+x+x^2+\cdots+x^{35})^5$

#2(c). $A(x) = (x^2+x^3+\cdots+x^{35})^5 = x^{10}(1+x+\cdots+x^{33})^5$

9.1 #3(a) $A(x) = (1+x+x^2+\cdots+x^{10})^6$. We want $[x^{10}]A(x)$.

#3(b) We want $[x^r]A(x)$ where
$A(x) = (1+x+\cdots+x^r)^n$ or $A(x) = (1+x+x^2+\cdots)^n$

9.2 #1(d) $A(x) = 0+0x+0x^2+6x^3-6x^4+6x^5-6x^6+\cdots$
$= 6x^3(1-x+x^2-x^3+\cdots)$

Using $B(x) = 1+x+x^2+x^3+\cdots = 1/(1-x)$ we have
$B(-x) = 1-x+x^2-x^3+\cdots = 1/(1-(-x)) = 1/(1+x)$ so
$A(x) = 6x^3/(1+x)$.

9.2 #1(e) $A(x) = 1+x^2+x^4+x^6+\cdots$
Using $B(x) = 1+x+x^2+x^3+\cdots = 1/(1-x)$ again
$A(x) = B(x^2) = 1/(1-x^2)$.

9.2 #2(b) $f(x) = x^4/(1-x) = x^4(1+x+x^2+\cdots) = x^4+x^5+x^6+\cdots$
So the sequence is $0,0,0,0,1,1,1,1,\ldots$

#2(d) $f(x) = 1/(1+3x)$.
Using $A(x) = 1+x+x^2+x^3+\cdots = 1/(1-x)$
$f(x) = A(-3x) = 1-3x+9x^2-27x^3+81x^3-\cdots$
So the sequence is $1,-3,3^2,-3^3,\ldots,(-3)^n,\ldots$

#2(e) $f(x) = 1/(3-x) = \frac{1}{3}/(1-\frac{x}{3})$.
$f(x) = \frac{1}{3}A(\frac{x}{3}) = \frac{1}{3}[1+\frac{1}{3}x+\frac{1}{9}x^2+\cdots]$ so
the sequence is $3^{-1}, 3^{-2}, 3^{-3}, \ldots, 3^{-n-1}, \ldots$