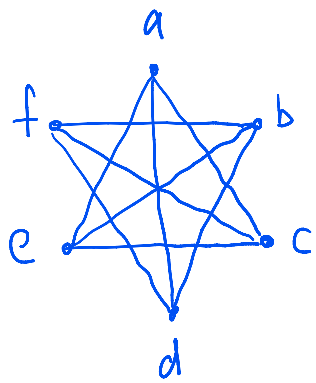
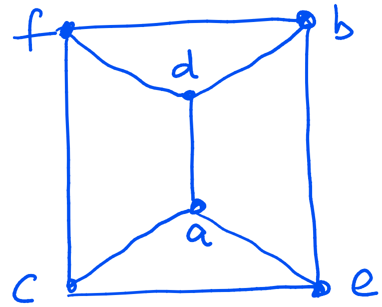
A7S21 Text Solutions

November 11, 2020 10:46 PM

11.4 14(b)



A planar drawing is



This is the prism graph.

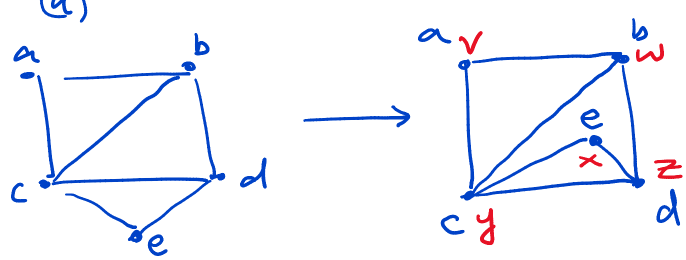
11.4 19 Let $G=(V,E)$. We need to use the formula

$$\sum_{v \in V} \deg(v) = 2|E|$$

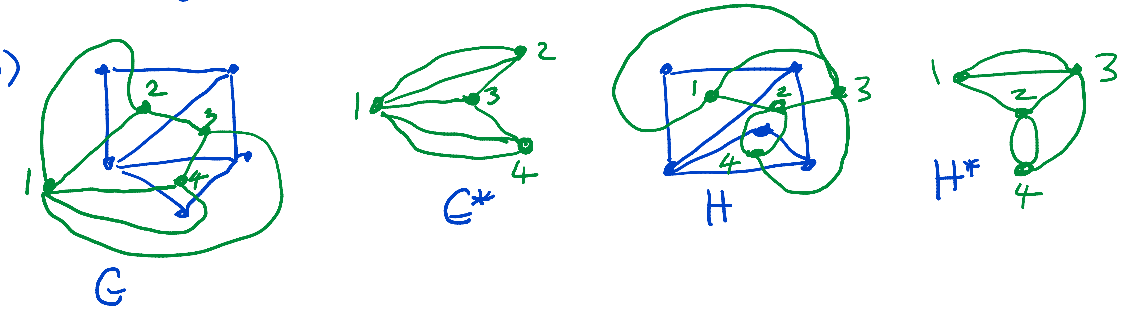
We are told $|E|=16$ and G is 4-regular which means $\deg v = 4$.
So $4|V| = 2 \cdot 16 \Rightarrow |V| = 8$. Now using Euler's Theorem

$$2 = |V| - |E| + |F| = 8 - 16 + |F| \Rightarrow |F| = 10.$$

11.4 #26 (a)



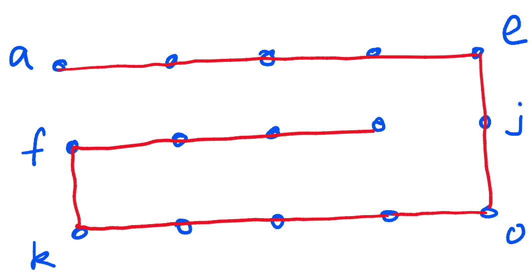
(b)

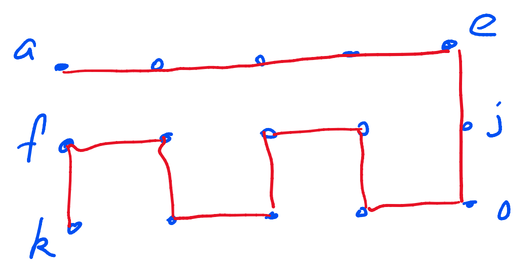


(c) G^* is not isomorphic to H^* since G^* has a vertex of degree 5 (G has 5 edges on the boundary)

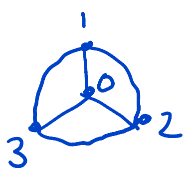
(c) G^* is not isomorphic to H^* since G^* has a vertex of degree 5 (G has 5 edges on the boundary) but H^* doesn't (H has 4 edges on its boundary)

11.5 3(e) There is no Hamiltonian Cycle but there are Hamiltonian paths. Here are two



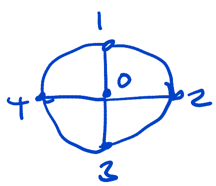


11.5 #6. How many different cycles are in W_n ?
I tried W_3 and W_4 to look for a pattern.
Let's start the cycle at the middle vertex 0.

$W_3 =$ 

0 1 2 3 0	0 1 3 2 0
0 2 3 1 0	0 2 1 3 0
0 3 1 2 0	0 3 2 1 0

So 3

$W_4 =$ 

0 1 2 3 4 0	0 1 4 3 2 0
0 2 3 4 1 0	0 2 1 4 3 0
0 3 4 1 2 0	0 3 2 1 4 0
0 4 1 2 3 0	0 4 3 2 1 0

So 4

So for W_n there are n H.C.s. where the first edge is $\{0, i\}$ for $1 \le i \le n$:

0 1 2 3 ⋯ n 0
0 2 3 ⋯ n 1 0
0 3 4 ⋯ n 1 2 0
⋮
0 n 1 2 ⋯ n−1 0

12.1 #3 We need the formula $|E| = |V| - K$
for a forest $G=(V,E)$ with $K \ge 1$ trees.

(a) $|E_1| = |V_1| - K_1 \Rightarrow |V_1| = |E_1| + K_1 = 40 + 7 = \underline{\underline{47}}$

(b) $|E_2| = |V_2| + K_2 \Rightarrow K_2 = |E_2| - |V_2| = 62 - 51 = \underline{\underline{11}}$

12.1 #10. If the connected undirected graph $G=(V,E)$ has 30 edges, what is the maximum value for $|V|$?
From $|V| = |E|+1$ a tree with
with 31 vertices has 30 edges and is connected. We cannot connect a graph with ≥ 32 vertices with 30 edges since the first edge connects two vertices and each subsequent edge connects one new vertex with one of the vertexes already connected. So 30 edges can only connect 31 vertices.
So 31 is the maximum.