

Due Date: Wednesday November 4, 2020, 11:59pm

Instructions

Answer all questions on paper or a tablet using your own handwriting. Put your name, student ID number and page number at the top of each page. If you use paper make a photo of each page and upload your solutions to crowdmark. If you use a tablet, export your assignment to .pdf and upload the .pdf to crowdmark.

Textbook Reading

- Sections: 9.1, 9.2, 9.3

Definitions, Concepts & Keywords

- Construct a generating function for a counting application.
- Find a formula for $[x^n]A(x)$ where $A(x)$ is a rational GF.
- Use Wolfram Alpha or Maple to extract a coefficient of a GF.

Exercises

A. Textbook Questions

- 9.1 Exercises 2ac, 3.
- 9.2 Exercises 1de, 2bde.
- 9.3 Exercises 2ab, 3.

B. Instructor Questions

Questions on 9.1

1. Execute the following command in Wolfram Alpha:
`Coefficient[(x+x^2+x^3+x^4+x^5+x^6)^6, x, 18]`
 to compute the coefficient of x^{18} in the polynomial $(x + x^2 + x^3 + x^4 + x^5 + x^6)^6$.
 Alternatively, if you have Maple, use the command
`coeff((x+x^2+x^3+x^4+x^5+x^6)^6, x, 18)`
2. For each equation, express the number of integer solutions as the coefficient of a polynomial. Then use Wolfram Alpha or Maple to calculate the coefficient.
 - (a) $a_1 + a_2 + a_3 = 14$ where $a_1, a_2, a_3 \geq 0$.
 - (b) $b_1 + b_2 + b_3 = 15$ where $2 \leq b_1 \leq 6$, b_2 is even and b_3 is odd.

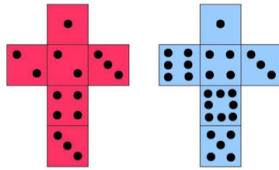


3. If a fair die is rolled 5 times, how many ways can the sum of the rolls equal 15? Use a generating function. Use Wolfram Alpha or Maple to compute the required coefficient.
4. Carol is collecting money from her cousins for a walkathon. Three cousins promise to give her either \$2, \$3, or \$4 and one promises to give her either \$5 or \$10. Let a_n be the number of ways she can get \$ n . Give a generating function for a_n . How many ways can she get \$15?

Questions on 9.2

5. Let $A(x) = 1 - x + x^2 - x^3 + x^4 - \dots$ and $B(x) = 0 + 1x + 2x^2 + 3x^3 + \dots$. Calculate rational GFs for $A(x)$, $B(x)$, $A(x) + B(x)$ and $B'(x)$.
6. Let $A(x) = 1 - x + x^2 - x^3 + x^4 - \dots$. Let $c_n = [x^n]A(x)^2$. Calculate a formula for c_n .
7. Let a_n be the number of ways to select n balls from a large bag of red, blue, and yellow balls where the selection must include an even number of blue balls. Write down a generating function for a_n in closed form.

8. Consider the set of six sided die where one dice has the numbers 1, 3, 4, 5, 6, 8 and the other has the numbers 1, 2, 2, 3, 3, 4 (see figure).



If these two die are rolled at the same time we'll investigate the possible sums and their frequencies in two ways: by direct enumeration in (a) and (b) below, and by using generating functions in (c) and (d) below.

- Determine the numbers which can occur as the sum of rolling these two dice.
 - For each number in (a) determine the number of possible ways this sum could be rolled. From this information, write down the generating polynomial $D(x)$ for the sum when rolling these two die.
 - Now, write down the generating polynomial $D_1(x)$ for the first dice, and $D_2(x)$ for the second dice.
 - Verify that $D_1(x) \cdot D_2(x) = D(x)$ by multiplying the two polynomials in part (c).
 - How does the generating polynomial $D(x)$ compare to the one for the sum when rolling two standard six sided die?
9. Determine a formula for the coefficient of x^n for the rational GFs $1/(1+2x)^3$ and $x/(2-x)^3$.
10. Find a rational GF for the sequence $0, 1, -2, 4, -8, 16, \dots$.
11. Consider the generating function $A(x) = \frac{1}{(2-x)(2+x)}$. Find a formula for $[x^n]A(x)$. Note, I get $[x^4]A(x) = 1/64$.
12. Let $A(x) = \sum_{n=0}^{\infty} a_n x^n = \frac{1}{(x-1)^2(2-x)}$. Find a formula for the coefficient a_n . Note, I get $a_2 = 17/8$.

Questions on 9.3

- List all partitions of 7.
- Find the generating function for the number of integer solutions of $2x + 3y + 5z = n$ for the cases
 - $x, y, z \geq 0$.
 - $x, y, z \geq 1$.