Due Date: Monday February 1st, 2021, 11pm

Instructions

Answer all questions on paper or a tablet using your own handwriting. Please number each page. Include a cover page with your name, student ID number and a list of the questions you have answered.

Do not upload my assignment to chegg. If you do and I catch you I will fail you. Michael Monagan.

Textbook Reading

- Sections: 11.2, 3.4, 3.5, 3.6

Exercises

A. **Textbook Questions**

Section 11.1 Exercise 10.

Section 11.2 Exercise 9. One pair of graphs is isomorphic and the other pair is not isomorphic. For the isomorphic pair, give an isomorphism, that is, give a bijection from the vertices of the first graph to the second.

Section 3.4 Exercises 7 and 12.

Section 3.5 Exercises 2 and 12 (omit $Pr(A \triangle B)$).

Section 3.6 Exercises 2 and 8.

B. **Instructors Questions**

1. For a positive integer n, what is the value of the counter after the following code has been executed. (Both C and Python code are included, use the language you are familiar with.)

C code:

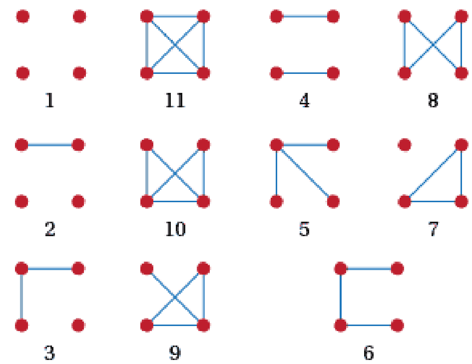
```c
int i,j,k,counter;
counter = 0;
for( i=1; i<=n; i++ )
   for( j=1; j<i; j++ )
       for( k=1; k<j; k++ )
           counter ++;
```

python code:

```python
counter = 0
for i in range(1,n+1):
   for j in range(1,i):
       for k in range(1,j):
           counter = counter + 1
```

2. Consider the graph G with vertices $V = \{1, 2, 3, 4, 5, 6\}$ and edges $E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{5, 6\}, \{2, 4\}\}$.
   (a) Draw G.
   (b) Is G bipartite? Explain.
   (c) List all spanning subgraphs of G that are trees.
3. Up to isomorphism, find all unlabelled graphs on 5 vertices with 4 edges. You should get six graphs. Identify which of your graphs are trees.
4. Read Definition 11.12 for the complement $\bar{G}$ of a graph G on page 523 of the textbook.
   (a) Draw the complement of $K_{1,1}$, $K_{2,2}$, and $K_{3,3}$.
   (b) Find a subgraph G of K_4 that is isomorphic to $\bar{G}$.
   (c) Why can't K_6 have a subgraph G that is isomorphic to $\bar{G}$?
5. Let n and k be integers with $3 \leq k \leq n$. How many cycles of length k are there in the complete graph K_n?
6. (a) For $n \geq 6$ how many subgraphs of K_n are isomorphic to $K_{2,4}$?
   (b) For $n \geq 4$ how many subgraphs of $K_{n,n}$ are isomorphic to $K_{2,4}$?

7. Shown in the figure below are all (unlabelled) graphs on 4 vertices.



The number of subgraphs of K_n isomorphic to the graph 2 is $\binom{n}{2}$ since every pair vertices in K_n with an edge between them gives one subgraph of K_n isomorphic to graph 2.

(a) How many subgraphs of K_n are isomorphic to graph 4?

(b) How many subgraphs of K_n are isomorphic to graph 9?

8. Suppose 6 men and 6 women are invited by a host to a dinner party.
Suppose the host selects 4 of the 12 guests at random for a game.

(a) What is the probability the host chooses two men and two women?

(b) What is the probability the host chooses one man and three women?

Do this by defining the sample space S and calculating $\Pr(A) = |A|/|S|$ where A is the set of considered outcomes.

9. A standard deck of 52 cards has 13 kinds of cards labelled $A, K, Q, J, 10, 9, 8, 7, 6, 5, 4, 3, 2$ in each of four suits ♥ (hearts), ♦ (diamonds) ♣ (clubs) and ♠ (spades). In the game of poker a player is dealt 5 cards from a standard deck of cards.

(a) What is the probability of being dealt 5 cards from one suit?
For example, 2♥, 7♥, 9♥, J♥, K♥

(b) What is the probability of being dealt a pair?
An example of a pair is J♥, 7♦, 9♥, 9♠, K♣

(c) What is the probability of being dealt two pairs?
An example of two pairs is 7♥, 7♦, 9♥, 9♠, K♣

10. Let S be a finite sample space and A, B and C be subsets of S. In class we showed that the $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$. Generalize this to find a formula for $Pr(A \cup B \cup C)$.

11. Let S be a sample space and A, B, C be events in S. Use the result from the previous question to determine $\Pr(B)$ given the following information

- A and B are independent,
- B and C are independent,
- A and C are disjoint, that is, $A \cap C = \emptyset$,
- $\Pr(A) = 1/5$,
- $\Pr(C) = 2/5$ and
- $\Pr(A \cup B \cup C) = 4/5$.

Hint: draw a Venn diagram with $A \cap C = \emptyset$.

12. A US company has developed an antibody test for whether a person has had COVID. If a person has had COVID, the probability that the test is positive is 0.98. If a person has NOT had COVID, the probability that the test if positive is 0.02. Suppose 10% of Americans have had COVID. Use Bayes' theorem to determine the probability that an American who tests positive has had COVID.