A1S21 Text Solutions

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1.2 Exercise 7. Each of the 9 choices catsup, mustard, mayo, lettuce tomato, onion, pickles, cheese and mushrooms may be chosen or not chosen. So there are $2^9 = 512$ hamburger orders. This is equivalent to the number of binary strings of length 9.

1.2 Exercise 10. Labelling the books $B_1, B_2, \ldots, B_{15}$, two possible arrangements are (I recommend you draw examples to get insight into the problem).

Arrangement 1

$B_1\ B_3\ B_5\ B_7\ B_9\ B_{11}\ B_{13}\ B_{15}$	Shelf 1
$B_2\ B_4\ B_6\ B_8\ B_{10}\ B_{12}\ B_{14}$	Shelf 2.

Arrangement 2

$B_1\ B_2\ B_3\ B_4\ B_5\ B_6\ B_7\ B_8$
$B_9\ B_{10}\ B_{11}\ B_{12} B_{13}\ B_{14}\ B_{15}$

Arrangement 1 has 8 books on shelf 1 and 7 books on shelf 2. We can take any one of 15! permutations of the 15 books to get a different arrangement with 8 books on shelf 1 and 7 on shelf 2, such as arrangement 2. Since there are 14 ways to divide 15 books into two groups with at least one book in each group, by the rule of product there are $14 \cdot 15!$ arrangements in total.

1.2 Exercise 15. How many ways can the symbols a, b, c, d, e, e, e, e, e be arranged so that there are no adjacent e's. There are 5 e's and 4 other symbols so each arrangement must look like

non-e

e _ e _ e _ e _ e

Every permutation of $\Sigma = \{a, b, c, d\}$ gives one arrangement e.g. the permutation bcda gives ebecedeae.
There are 4! permutations of $\Sigma = \{a, b, c, d\}$ so 4! arrangements.

1.3 Exercise 4

(a) There are six "dots" which can be raised or not raised. So $2^6 = 64$ possible symbols can be represented. However, since at least one dot must be raised this rules out one case so $2^6 - 1 = 63$ symbols can be represented.

(b) How many symbols have 3 raised dots? Answer $\binom{6}{3} = 20$.

(c) How many symbols have an even # of dots raised?

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Answer $\binom{6}{2}+\binom{6}{4}+\binom{6}{6}= 2\cdot\binom{6}{2}+1= 2\cdot 15+1=31$.

1.3 Exercise 30.

Let $S = \binom{n}{0}+2\binom{n}{1}+2^2\binom{n}{2}+2^3\binom{n}{3}+\cdots+2^k\binom{n}{k}+\cdots+2^n\binom{n}{n}$

Consider $(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}$. Let $x=2$ and $y=1$.

Then $(2+1)^n = \sum_{k=0}^{n}\binom{n}{k}2^k\cdot 1^{n-k} = \sum_{n=0}^{n} 2^k\binom{n}{k}\cdot 1 = S$.

Therefore $S=3^n$.

1.4 Exercise 2. Let x_i be the number of candy bars child i gets.

Then $x_1+x_2+x_3+x_4+x_5 = 15$.

We are told the youngest child say child 1 gets 1 or 2 candy bars.

So we have <u>two</u> cases:

Case $x_1=1 \Rightarrow 1+x_2+x_3+x_4+x_5=15$ with $x_i\geq 0$

$\Rightarrow x_2+x_3+x_4+x_5=14$ with $x_i\geq 0$.

There are $\binom{14+4-1}{14}=\binom{17}{14}$ ways to distribute the candy bars.

Case $x_1=2 \Rightarrow 2+x_2+x_3+x_4+x_5=15$ with $x_i\geq 0$

$\Rightarrow x_2+x_3+x_4+x_5=13$ with $x_i\geq 0$

There are $\binom{13+4-1}{13}=\binom{16}{13}$ ways in this case.

The total number of ways is $\binom{17}{14}+\binom{16}{13}$.

1.4 Exercise 12. Determine the number of integer solutions to

$$x_1+x_2+x_3+x_4+x_5 < 40$$

Where (a) $x_i\geq 0$ for $1\leq i\leq 5$.

This is the same as the number of integer solutions to

$$x_1+x_2+x_3+x_4+x_5+y=39 \quad \text{where } x_i\geq 0 \text{ and } y\geq 0$$

which is $\binom{39+6-1}{39}=\binom{44}{39}$

Where (b) $x_i\geq -3$ for $1\leq i\leq 5$

This is the same as the number of integer solutions to where (b) $x_i \geq -3$ for $1 \leq i \leq 5$

This is the same as the number of integer solutions to

$$x_1 + x_2 + x_3 + x_4 + x_5 + y = 39 + 15 \quad \text{where } x_i \geq 0 \text{ and } y \geq 0$$

which is $\binom{54+6-1}{54} = \binom{59}{54}$.

1.4 Exercise 15. In how many ways can Beth place 24 <u>different</u> books on 4 bookshelves with at least one book on each shelf.

This is similar to 1.2 Exercise 10. Let b_1, b_2, b_3, b_4 be the number of books on the four shelves. We require

$$b_1 + b_2 + b_3 + b_4 = 24 \quad \text{with } b_i \geq 1.$$

The number of integer solutions is determined by

$$x_1 + x_2 + x_3 + x_4 = 20 \quad \text{with } x_i \geq 0.$$

This has $\binom{20+4-1}{20} = \binom{23}{20}$ ways. Here is one way

B1 B2 B3 B4 B5 B6 B7 B8 B9 B10	Shelf 1
B11 B12 B13 B14	Shelf 2
B15	Shelf 3
B16 B17 B18 B19 B20 B21 B22 B23 B24	Shelf 4

We can permute each such arrangement in $24!$ ways. So the total number of arrangements is $\binom{23}{20} \cdot 24!$