A3 textbook solutions

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3.7 Exercise 2. We are given $Pr(X=x) = (3x+1)/22$ for $x = 0,1,2,3$.

Let's enumerate $Pr(X=x)$:

x	0	1	2	3.
$Pr(X=x)$	$1/22$	$4/22$	$7/22$	$10/22$

(a) $Pr(X=3) = 10/22$

(b) $Pr(X \leq 1) = 4/22 + 1/22 = 5/22$

(c) $Pr(1 \leq X \leq 3) = 1 - Pr(X=0) = 1 - \frac{1}{22} = 21/22$

(e) Let A be the event that $X=1$ $\quad Pr(A) = 4/22$

Let B be the event that $X \leq 2$ $\quad Pr(B) = 12/22$.

$$Pr(X=1 \mid X \geq 2) = Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A)}{Pr(B)} = \frac{4/22}{12/22} = \frac{1}{3}$$

Because $A \subset B$ we have $Pr(A \cap B) = Pr(A)$.

(d) $Pr(X > -2) = 1$.

10.1 Exercise 1

a) The sequence $2, 10, 50, 250, \ldots$ is multiplying by 5 so $a_n = 5a_{n-1}$ and $a_1 = 2$.

b) The sequence $6, -18, 54, -162, \ldots$ is multiplying by -3 so $a_n = -3a_{n-1}$ and $a_1 = 6$.

c) The sequence $7, 14/5, 28/25, 56/125, \ldots$ is multiplying by $\frac{2}{5}$ so $a_n = \frac{2}{5}a_{n-1}$ and $a_1 = 7$.

10.1 Exercise 2.

(a) $a_{n+1} - 1.5a_n = 0 \Rightarrow a_{n+1} = 1.5a_n$

$\Rightarrow a_n = A\, 1.5^n$ for some constant A

(b) $4a_n - 5a_{n-1} = 0 \Rightarrow a_n = \frac{5}{4}a_{n-1} \Rightarrow a_n = A \cdot \left(\frac{5}{4}\right)^n$.

(c) $3a_{n+1} - 4a_n = 0 \Rightarrow a_{n+1} = \frac{4}{3}a_n \Rightarrow a_n = A \cdot \left(\frac{4}{3}\right)^n$.

$a_1 = 5 = A \cdot \left(\frac{4}{3}\right)^1 \Rightarrow A = 15/4 \Rightarrow a_n = \frac{15}{4}\left(\frac{4}{3}\right)^n$

(d) $2a_n - 3a_{n-1} = 0 \Rightarrow a_n = \frac{3}{2}a_{n-1} \Rightarrow a_n = A\cdot\left(\frac{3}{2}\right)^n.$

$a_4 = 81 = A\cdot\left(\frac{3}{2}\right)^4 = A\cdot\frac{81}{16} \Rightarrow A = 16$

Thus $a_n = 16\left(\frac{3}{2}\right)^n.$

10.2 Exercise 1. We need to calculate the solutions to the characteristic equation $ar^2+br+c=0$. Factoring is one way. Using the quadratic formula $r = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ works too.

(a) $a_n = 5a_{n-1} + 6a_{n-2}$ for $n \geq 2$ $a_0 = 1$ $a_1 = 3.$

$\Rightarrow a_n - 5a_{n-1} - 6a_{n-2} = 0$

$\Rightarrow r^2 - 5r - 6 = (r-6)(r+1) = 0 \Rightarrow r = -1, 6.$

The general solution is $a_n = A(-1)^n + B\,6^n$

$a_0 = 1 = A + B$ (1)

$a_1 = 3 = -A + 6B$ (2)

Solving these equations I get $4 = 7B \Rightarrow B = \frac{4}{7}, A = \frac{3}{7}$

$a_n = \frac{3}{7}(-1)^n + \frac{4}{7}6^n$

(b) $2a_{n+2} - 11a_{n+1} + 5a_n = 0$ for $n \geq 0$ $a_0 = 2,$ $a_1 = -8$

$\Rightarrow 2a_n - 11a_{n-1} + 5a_{n-2} = 0$ for $n \geq 2$

$\Rightarrow 2r^2 - 11r + 5 = (2r-1)(r-5) = 0 \Rightarrow r = \frac{1}{2}, 5$

The general solution is $a_n = A\left(\frac{1}{2}\right)^n + B\,5^n$

$a_0 = A + B = 2$

$a_1 = A/2 + 5B = -8$

$\Rightarrow A + 10B = -16$

$\Rightarrow 9B = -18 \Rightarrow B = -2 \Rightarrow A = 4.$

Thus $a_n = 4/2^n - 2\cdot 5^n$

(d) $a_n - 6a_{n-1} + 9a_{n-2} = 0$ for $n \geq 2$ $a_0 = 5,$ $a_1 = 12.$

$r^2 - 6r + 9 = (r-3)^2 \Rightarrow r = 3, 3.$

The general solution is

$a_n = A\,3^n + Bn\,3^n$

$a_0 = A + 0 = 5$

$a_1 = 3A + 3B = 12 \Rightarrow 15 + 3B = 12 \Rightarrow B = -1.$

So $a_n = 5\cdot 3^n - 3^n\cdot n = 3^n(5-n).$

$a_1 = 3A + 3B = 12 \Rightarrow 15 + 3B = 12 \Rightarrow B = -1.$

So $a_n = 5 \cdot 3^n - 3^n \cdot n = 3^n(5-n)$.

10.2 Exercise 4.

Let a_n be the # ways to park the motorcycles (Ms) and compact cars (Cs) in n spaces.

$a_1 = 1$ $\quad a_2 = 2$ as we can have 2Ms or 1C.

$a_3 = 3$ as we can have MMM or MC or CM.

For a_n we can have $MM\underbrace{\quad}_{a_{n-2}}$ or $C\underbrace{\quad}_{a_{n-1}}$

thus $a_n = a_{n-2} + a_{n-1}$ which is the Fibonnaci recurrence.

We have $r^2 - r - 1 = 0 \Rightarrow r = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1}{2} + \frac{\sqrt{5}}{2}, \frac{1}{2} - \frac{\sqrt{5}}{2}$

Thus $a_n = A\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)^n + B\left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)^n$.

Using $a_0 = 1 = A + B$

$a_1 = 1 = \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)A + \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)B$

I get $A = \frac{1}{2} + \frac{\sqrt{5}}{10}$, $B = \frac{1}{2} - \frac{\sqrt{5}}{10}$.