Due Date: Wednesday November 4, 2020, 11:59pm

Instructions

Answer all questions on paper or a tablet using your own handwriting. Put your name, student ID number and page number at the top of each page. If you use paper make a photo of each page and upload your solutions to crowdmark. If you use a tablet, export your assignment to .pdf and upload the .pdf to crowdmark.

Textbook Reading

- Sections: 9.1, 9.2

Definitions, Concepts & Keywords

- Construct a generating function for a counting application.
- Find a formula for $[x^n]A(x)$ where $A(x)$ is a rational GF.
- Use Wolfram Alpha or Maple to extract a coefficient of a GF.

Exercises

A. **Textbook Questions**

9.1 Exercises 2ac, 3.

9.2 Exercises 1de, 2bde.

B. **Instructor Questions**

Questions on 9.1

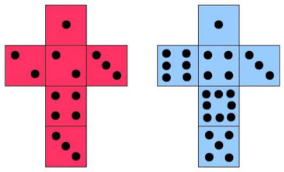
1. Execute the following command in Wolfram Alpha:
   `Coefficient[ (x+x^2+x^3+x^4+x^5+x^6)^6, x, 18 ]`
   to compute the coefficient of x^{18} in the polynomial $(x + x^2 + x^3 + x^4 + x^5 + x^6)^6$.
   Alternatively, if you have Maple, use the command
   `coeff( (x+x^2+x^3+x^4+x^5+x^6)^6, x, 18 )`
2. For each equation, express the number of integer solutions as the coefficient of a polynomial. Then use Wolfram Alpha or Maple to calculate the coefficient.
   (a) $a_1 + a_2 + a_3 = 14$ where $a_1, a_2, a_3 \geq 0$.
   (b) $b_1 + b_2 + b_3 = 15$ where $2 \leq b_1 \leq 6$, b_2 is even and b_3 is odd.

3. If a fair die is rolled 5 times, how many ways can the sum of the rolls equal 15?
   Use a generating function. Use Wolfram Alpha or Maple to compute the required coefficient.
4. Carol is collecting money from her cousins for a walkathon. Three cousins promise to give her either \$2, \$3, or \$4 and one promises to give her either \$5 or \$10. Let a_n be the number of ways she can get \$$n$. Give a generating function for a_n. How many ways can she get \$15?

Questions on 9.2

5. Let $A(x) = 1 - x + x^2 - x^3 + x^4 - \dots$ and $B(x) = 0 + 1x + 2x^2 + 3x^3 + \dots$.
   Calculate rational GFs for $A(x), B(x), A(x) + B(x)$ and $B'(x)$.
6. Let $A(x) = 1 - x + x^2 - x^3 + x^4 - \dots$. Let $c_n = [x^n]A(x)^2$. Calculate a formula for c_n.
7. Let a_n be the number of ways to select n balls from a large bag of red, blue, and yellow balls where the selection must include an even number of blue balls. Write down a generating function for a_n in closed form.

8. Consider the set of six sided die where one dice has the numbers 1, 3, 4, 5, 6, 8 and the other has the numbers 1, 2, 2, 3, 3, 4 (see figure).



If these two die are rolled at the same time we'll investigate the possible sums and their frequencies in two ways: by direct enumeration in (a) and (b) below, and by using generating functions in (c) and (d) below.

   (a) Determine the numbers which can occur as the sum of rolling these two dice.
   (b) For each number in (a) determine the number of possible ways this sum could be rolled. From this information, write down the generating polynomial $D(x)$ for the sum when rolling these two die.
   (c) Now, write down the generating polynomial $D_1(x)$ for the first dice, and $D_2(x)$ for the second dice.
   (d) Verify that $D_1(x) \cdot D_2(x) = D(x)$ by multiplying the two polynomials in part (c).
   (e) How does the generating polynomial $D(x)$ compare to the one for the sum when rolling two standard six sided die?

9. Determine a formula for the coefficient of x^n for the rational GFs $1/(1+2x)^3$ and $x/(2-x)^3$.
10. Find a rational GF for the sequence $0, 1, -2, 4, -8, 16, \ldots$.
11. Consider the generating function $A(x) = \dfrac{1}{(2-x)(2+x)}$. Find a formula for $[x^n]A(x)$. Note, I get $[x^4]A(x) = 1/64$.
12. Let $A(x) = \sum_{n=0}^{\infty} a_n x^n = \dfrac{1}{(x-1)^2(2-x)}$. Find a formula for the coefficient a_n. Note, I get $a_2 = 17/8$.
13. Let $A(x) = 1/(1+x+x^2) = \sum_{n=0}^{\infty} a_n x^n$.
    To find $A(x)$ solve $A(x)(1+x+x^2) = 1$ for $a_0, a_1, \ldots, a_4$ then find a recurrence for a_n.
14. Let $A(x) = x/(1-x-x^2-x^3) = \sum_{n=0}^{\infty} a_n x^n$.
    Find $A(x)$ up to x^5 and find a recurrence for a_n.