Theorem 11.7 (Trager – Rothstein ~1976)

Let $C, D \in K[x]$, $0 \leq \deg C < \deg D$, $\gcd(C,D)=1$, $\operatorname{lc} D = 1$.
Let $R(z) = \operatorname{res}(C - zD', D, x) \in K[z]$
Then $\int \frac{C}{D}\,dx = \alpha_1 \ln(v_1) + \cdots + \alpha_k \ln(v_k)$ where

(i) α_i are the distinct roots of $R(z)$ and
(ii) $v_i = \gcd(C - \alpha_i D', D)$ (monic gcd)

Note if $L = K(\alpha_i)$ then $v_i \in L[x]$.

Theorem 11.8 (restated)
Suppose F is a field and $\int \frac{C}{D}\,dx = \sum \beta_i \ln(w_i)$ where $\beta_i \in F$ and $w_i \in F[x]$. Then $F \supset L$.