MATH 340 Assignment 3, Fall 2010

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This assignment is to be handed in on Tuesday October 12th by 11:30am. Late penalty: -20% for handing in up to 24 hours late. Zero after that.

Section 1.7: Equations in \mathbb{Z}_n

Exercise 14.

Prove Corollary 1.7.6 parts (i) and (ii) on page 39. Note that our textbook uses (a, b) = 1 to mean gcd(a, b) = 1.

Which elements of \mathbb{Z}_{24} are units and which are zero-divisors?

Section 1.10: Euler's ϕ -function

Exercise 3, 4.

Section 1.11: Theorem's of Euler and Fermat

Exercises 2, 3, 8.

Prove Theorem 1.11.1 (Euler's theorem) using the same approach given in class to prove Theorem 1.11.3 (Fermat's little Theorem). First prove the Lemma: if $a \in \mathbb{Z}_n^*$ then $a\mathbb{Z}_n^* = \mathbb{Z}_n^*$ where \mathbb{Z}_n^* denotes the set of units in \mathbb{Z}_n .

Section 2.1: Basic Properties

Exercises 1, 4, 7, 10, 14, 16, 17. For exercise 10, use the fact that $\mathbb{Q}[\sqrt{2}]$ is a subset of the real numbers \mathbb{R} which is a field,

so that you don't have to prove all the field axioms. For exercise 17, give also an example of a zero divisor.