MATH 340 Assignment 8, Fall 2010

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This assignment is due Monday December 6th at 11:20 am. Late penalty: -20% for up to 24 hours late. Zero after that. For problems requiring Maple please submit a printout of a Maple worksheet.

Section 2.13: Isomorphisms Between (finite) Fields

Exercises 1, 6(i), 10 and prove Lemma 2.13.1 part (ii).

For exercise 1, factor the polynomial $f(x) = x^3 + x + 1$ over F and over G. It should factor into a product of linear factors over F and over G. Using Maple, check that the isomorphism $\phi: F \to G$ that you find satisfies (i) ϕ is a bijection, (ii) $\phi(a+b) = \phi(a) + \phi(b)$ and (iii) $\phi(a \cdot b) = \phi(a) \cdot \phi(b)$ for all $a, b \in F$.

Section 2.14: Error Correcting Codes

Exercises 1, 4, 6, 8(i).

For exercise 8(i) use Maple and also determine d(C).

Section 3.1: Basic Properties (of Groups)

Exercises 1, 5, 7(iii), 8, 9, 18.

In question 7(iii) you will find a group with 4 elements. The group G is isomorphic to $\mathbb{Z}_4(+)$. Find an isomorphism $\phi: G(\cdot) \to \mathbb{Z}_4(+)$ that satisfies $\phi(a \cdot b) = \phi(a) + \phi(b)$. Note the group operation in G is multiplication but the group operation in $\mathbb{Z}_4(+)$ is addition.

For exercise 18, determine also the order of each element of D_3 .