MATH 152 98-2 Solutions for Assignment 1

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2. $f(x) = 3x - 1$; the partition P of $[-2, 2]$ is $\{-2, -1.2, -0.6, 0, 0.8, 1.6, 2\}$.
x_i^* is the midpoint of the $i^{\underline{th}}$ interval.

(a) $\|P\| = \text{Max}\,(0.8, 0.6, 0.6, 0.8, 0.8, 0.4) = 0.8.$

(b) The Riemann sum associated with this partition and this choice of $\{x_i^*\}$ is

$$\sum_{i=1}^{n} f(x_i^*)\,\Delta x_i = f(-1.6)\cdot 0.8 + f(-0.9)\cdot 0.6 + f(-0.3)\cdot 0.6 + f(0.4)\cdot 0.8 + f(1.2)\cdot 0.8 + f(1.8)\cdot 0.4 =$$

$$= -5.8\cdot 0.8 - 3.7\cdot 0.6 - 1.9\cdot 0.6 + 0.2\cdot 0.8 + 2.6\cdot 0.8 + 4.4\cdot 0.4 = -4.$$

4. $f(x) = x + x^2$; the partition P of $[-2, 0]$ is given by $\{-2, -1.5, -1, -0.7, -0.4, 0\}$.
x_i^* is the left endpoint of the $i^{\underline{th}}$ interval.

(a) $\|P\| = \text{Max}\,(0.5, 0.5, 0.3, 0.3, 0.4) = 0.5.$

(b) The Riemann sum associated with this partition and this choice of $\{x_i^*\}$ is

$$\sum_{i=1}^{n} f(x_i^*)\,\Delta x_i = f(-2)\cdot 0.5 + f(-1.5)\cdot 0.5 + f(-1)\cdot 0.3 + f(-0.7)\cdot 0.3 + f(-0.4)\cdot 0.4 =$$

$$= 2\cdot 0.5 + 0.75\cdot 0.5 + 0\cdot 0.3 - 0.21\cdot 0.3 - 0.24\cdot 0.4 = 1.216.$$

16. $$\int_{-2}^{7} (6 - 2x)\,dx = \lim_{n\to\infty} \frac{9}{n} \sum_{i=1}^{n} f(-2 + (9i)/n) = \lim_{n\to\infty} \frac{9}{n} \sum_{i=1}^{n} \left[10 - \frac{18i}{n}\right] =$$

$$= \lim_{n\to\infty} \frac{9}{n} \left[10n - \frac{18}{n}\cdot\frac{n(n+1)}{2}\right] = \lim_{n\to\infty}\left[90 - 81\left[1 + \frac{1}{n}\right]\right] = 9.$$

30. $$\lim_{\|P\|\to 0} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x_i = \int_1^4 \sqrt{x}\,dx$$ if the partition P is on the interval $[1, 4]$.

34. $$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{1 + (i/n)^2} = \int_0^1 \frac{dx}{1 + x^2}.$$
There are other correct answers.

36. $$\int_1^1 x^2 \cos x\,dx = 0.$$
See Note 6 on Page 338 of the text.

40. $$\int_3^6 (4 - 7x)\,dx = \int_3^6 4\,dx - 7\int_3^6 x\,dx = 4(6 - 3) - 7\cdot\frac{1}{2}(6^2 - 3^2) = -82.5.$$