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6. If $g(x) = \int_{-1}^{x} \sqrt{t^3+1}\,dt$ then $g'(x) = \sqrt{x^3+1}$.

10. If $F(x) = \int_{x}^{4} \left(2+\sqrt{u}\,\right)^8 du = -\int_{4}^{x} \left(2+\sqrt{u}\,\right)^8 du$ then $F'(x) = -\left(2+\sqrt{x}\,\right)^8$.

16. If $y = \int_{-5}^{\sin x} t\cos(t^3)\,dt$ then $\frac{dy}{dx} = \sin x \cdot \cos(\sin^3 x) \cdot \cos x$
by the Chain Rule and Part 1 of the Fundamental Theorem of Calculus.

20. $\int_{1}^{2} (5x^2 - 4x + 3)\,dx = \left(\frac{5}{3}x^3 - 2x^2 + 3x\right)\Big]_{1}^{2} = \frac{26}{3}$.

40. $\int_{-5}^{-2} \frac{x^4-1}{x^2+1}\,dx = \int_{-5}^{-2} (x^2-1)\,dx = \left(\frac{1}{3}x^3 - x\right)\Big]_{-5}^{-2} = 36$.

46. $\int_{\pi/4}^{\pi} \sec^2\theta = \tan\theta\Big]_{\pi/4}^{\pi} = -1$ is **incorrect**.
Integrating from left to right, how could a positive integrand yield a negative integral?
The Fundamental Theorem of Calculus does not apply, because of the infinite discontinuity of $\sec^2\theta$ at $\theta = \frac{\pi}{2}$.
The integral $\int_{\pi/4}^{\pi} \sec^2\theta$ does not exist.

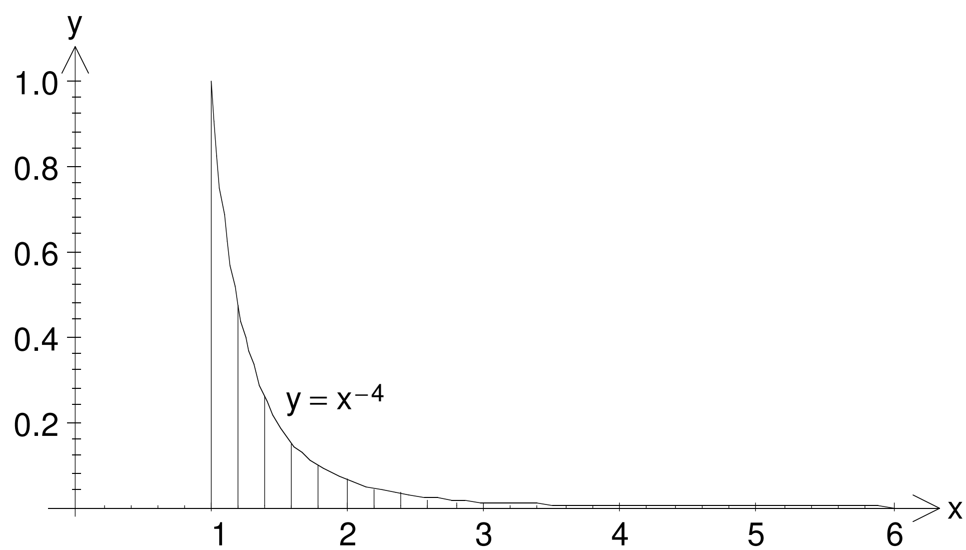
48. $\int_{\ln 3}^{\ln 6} 8e^x\,dx = 8e^x\Big]_{\ln 3}^{\ln 6} = 24$.

50. $\int_{-e^2}^{-e} \frac{3}{x}\,dx = 3\ln|x|\Big]_{-e^2}^{-e} = -3$.

52. $\int_{0}^{0.5} \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x\Big]_{0}^{0.5} = \frac{\pi}{6}$.

60. If $f(x) = \begin{cases} x & \text{if } -\pi \le x \le 0 \\ \sin x & \text{if } \quad 0 < x \le \pi \end{cases}$
then $\int_{-\pi}^{\pi} f(x)\,dx = \int_{-\pi}^{0} x\,dx + \int_{0}^{\pi} \sin x\,dx = \frac{1}{2}x^2\Big]_{-\pi}^{0} - \cos x\Big]_{0}^{\pi} = -\frac{1}{2}\pi^2 + 2$.

62. To the right is a graph of $y = x^{-4}$, $1 \le x \le 6$, with subdivisions at intervals of 0.2 along the x-axis and at intervals of 0.04 along the y-axis. Vertical line segments are drawn for $1 \le x \le 3.4$; after that the segments are too short to see.



Using my TI–36 calculator I found the sum of the areas of the trapezoids approximating the indicated vertical strips; the first trapezoid top for instance goes from (1, 1) to $(1.2, 1.2^{-4})$. The result I obtained is 0.338651206 which I will round off to 0.34. Notice that the tops of the trapezoids are outside the region since the curve is concave upwards, and this means I am overestimating the area inside each strip. I am neglecting the strips to the right of $x = 3.4$ (which don't contribute much area).

Area $= \int_1^6 x^{-4}\,dx = -\frac{1}{3}x^{-3}\Big]_1^6 = \frac{215}{648} \approx 0.331790123$. The approximation is rather good!

74. $\int (\cos x - 2\sin x)\,dx = \sin x + 2\cos x + C.$

78. (a) If $v(t) = (t^2 - 2t - 8)$ m/s, $1 \le t \le 6$ gives the velocity then the displacement is $s(6) - s(1) = \int_1^6 (t^2 - 2t - 8)\,dt = \left(\frac{1}{3}t^3 - t^2 - 8t\right)\Big]_1^6 = -\frac{10}{3}$ m.

(b) $v(t) = (t-4)(t+2)$ so $v(t) \le 0$ for $1 \le t \le 4$ and $v(t) \ge 0$ for $4 \le t \le 6$. The distance traveled is $\int_1^6 |t^2 - 2t - 8|\,dt = \int_1^4 (-t^2 + 2t + 8)\,dt + \int_4^6 (t^2 - 2t - 8)\,dt =$
$= \left(-\frac{1}{3}t^3 + t^2 + 8t\right)\Big]_1^4 + \left(\frac{1}{3}t^3 - t^2 - 8t\right)\Big]_4^6 = 18 + \frac{44}{3} = \frac{98}{3}$ m.

80. (a) If $a(t) = (2t + 3)$ m/s^2 and $v(0) = -4$ m/s, $0 \le t \le 3$, $v(t) = (t^2 + 3t - 4)$ m/s.

(b) $v(t) = (t+4)(t-1)$ so $v(t) \le 0$ for $0 \le t \le 1$ and $v(t) \ge 0$ for $1 \le t \le 3$. The distance traveled is $\int_0^3 |t^2 + 3t - 4|\,dt = \int_0^1 (-t^2 - 3t + 4)\,dt + \int_1^3 (t^2 + 3t - 4)\,dt =$
$= \left(-\frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t\right)\Big]_0^1 + \left(\frac{1}{3}t^3 + \frac{3}{2}t^2 - 4t\right)\Big]_1^3 = \frac{13}{6} + \frac{38}{3} = \frac{89}{6}$ m.

82. The increase in population is $\int_4^{10} (200 + 50t)\,dt = (200t + 25t^2)\Big]_4^{10} = 3300.$

86. If $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt$, then $g'(x) = \frac{1}{\sqrt{2+x^8}} \cdot 2x - \frac{1}{\sqrt{2+\tan^4 x}} \cdot \sec^2 x.$

To see why, write $g(x) = \int_a^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt - \int_a^{\tan x} \frac{1}{\sqrt{2+t^4}}\,dt$ for some constant a, and apply the Chain Rule and Part 1 of the Fundamental Theorem of Calculus.

44. $\int_3^4 f(x)\,dx + \int_1^3 f(x)\,dx + \int_4^1 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^4 f(x)\,dx + \int_4^1 f(x)\,dx =$
$= \int_1^4 f(x)\,dx + \int_4^1 f(x)\,dx = \int_1^4 f(x)\,dx - \int_1^4 f(x)\,dx = 0.$

48. $\int_{-3}^5 f(x)\,dx - \int_{-3}^0 f(x)\,dx + \int_5^6 f(x)\,dx = \int_{-3}^5 f(x)\,dx + \int_0^{-3} f(x)\,dx + \int_5^6 f(x)\,dx =$
$= \int_0^{-3} f(x)\,dx + \int_{-3}^5 f(x)\,dx + \int_5^6 f(x)\,dx = \int_0^5 f(x)\,dx + \int_5^6 f(x)\,dx = \int_0^6 f(x)\,dx.$

50. $\sqrt{5-x} \ge \sqrt{5-2} = \sqrt{3} = \sqrt{2+1} \ge \sqrt{x+1}$ if $1 \le x \le 2$.
Therefore $\int_1^2 \sqrt{5-x}\,dx \ge \int_1^2 \sqrt{x+1}\,dx$ by Property 7 on Page 343 of the text.

56. If $0 \le x \le 2$, then $1 \le \sqrt{x^3+1} \le 3$, so $1 \cdot (2-0) \le \int_0^2 \sqrt{x^3+1}\,dx \le 3 \cdot (2-0)$
by Property 8 on Page 343, so $2 \le \int_0^2 \sqrt{x^3+1}\,dx \le 6$.
The approximate value of the integral $\int_0^2 \sqrt{x^3+1}\,dx$ is 3.2413092631952725570, according to *Maple* on my Macintosh.
I do not know how to evaluate this integral exactly.

62. If $2 \le x \le 5$, then $\sqrt{x^2-1} \le x$, so $\int_2^5 \sqrt{x^2-1}\,dx \le \int_2^5 x\,dx = \frac{1}{2}(5^2 - 2^2) = 10.5.$
In fact the exact value of the integral $\int_2^5 \sqrt{x^2-1}\,dx$ is $5\sqrt{6} - \sqrt{3} + \frac{1}{2}\ln\frac{2+\sqrt{3}}{5+2\sqrt{6}}$,
which is approximately 10.027661020028832708.