MATHEMATICS 152 98-2 Solutions for Assignment 3

2. Let $u = 2 + x^3$, so $du = 3x^2\,dx$.
$\int \frac{x^2}{\sqrt{2+x^3}}\,dx = \int \frac{du}{3\sqrt{u}} = \frac{2}{3}\sqrt{u} + C = \frac{2}{3}\sqrt{2+x^3} + C.$

6. Let $u = a\theta$, so $du = a\,d\theta$.
$\int \sec(a\theta)\tan(a\theta)\,d\theta = \frac{1}{a}\int \sec u \tan u\,du = \frac{1}{a}\sec u + C = \frac{1}{a}\sec(a\theta) + C.$

8. Let $u = 1 - x^4$, so $du = -4x^3\,dx$.
$\int x^3(1-x^4)^5\,dx = -\frac{1}{4}\int u^5\,du = -\frac{1}{24}u^6 + C = -\frac{1}{24}(1-x^4)^6 + C.$

12. Let $u = x^2 + 1$, so $du = 2x\,dx$.
$\int x(x^2+1)^{3/2}\,dx = \frac{1}{2}\int u^{3/2}\,du = \frac{1}{5}u^{5/2} + C = \frac{1}{5}(x^2+1)^{5/2} + C.$

16. Let $u = 3 - 5y$, so $du = -5\,dy$.
$\int \sqrt[5]{3-5y}\,dy = -\frac{1}{5}\int u^{1/5}\,du = -\frac{1}{6}u^{6/5} + C = -\frac{1}{6}(3-5y)^{6/5} + C.$

24. Let $u = 1 - t^3$, so $du = -3t^2\,dt$.
$\int t^2\cos(1-t^3)\,dt = -\frac{1}{3}\int \cos u\,du = -\frac{1}{3}\sin u + C = -\frac{1}{3}\sin(1-t^3) + C.$

32. Let $u = x^3 + 1$, so $du = 3x^2\,dx$, and $x^5\,dx = (u-1)x^2\,dx = \frac{1}{3}(u-1)\,du$.
$\int \sqrt[3]{x^3+1}\,x^5\,dx = \frac{1}{3}\int u^{1/3}(u-1)\,du = \frac{1}{3}\int (u^{4/3} - u^{1/3})\,du = \frac{1}{7}u^{7/3} - \frac{1}{4}u^{4/3} + C =$
$= \frac{1}{7}(x^3+1)^{7/3} - \frac{1}{4}(x^3+1)^{4/3} + C.$

34. Let $u = \sin x$, so $du = \cos x\,dx$.
$\int \cos x\cos(\sin x)\,dx = \int \cos u\,du = \sin u + C = \sin(\sin x) + C.$

36. Let $u = x^2 + 1$, so $du = 2x\,dx$.
$\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+1| + C = \frac{1}{2}\ln(x^2+1) + C.$ (Note $x^2 + 1 > 0$.)

38. $\int x e^{x^2} dx = \frac{1}{2} e^{x^2} + C$. (Let $u = x^2$, $du = 2x\,dx$.)

40. $\int \frac{\tan^{-1} x}{1 + x^2}\, dx = \frac{1}{2} (\tan^{-1} x)^2 + C$. (Let $u = \tan^{-1} x$, $du = \frac{1}{1 + x^2}\, dx$.)

42. $\int e^x \sin(e^x)\, dx = -\cos(e^x) + C$. (Let $u = e^x$, $du = e^x dx$.)

46. $\int \frac{\sin x}{1 + \cos^2 x}\, dx = -\tan^{-1}(\cos x) + C$. (Let $u = \cos x$, $du = -\sin x\, dx$.)

70. Let $u = x^2 + a^2$, so $du = 2x\,dx$. When $x = -a$, $u = 2a^2$. When $x = a$, $u = 2a^2$.
$\int_{-a}^{a} x\sqrt{x^2 + a^2}\, dx = \frac{1}{2}\int_{2a^2}^{2a^2} u^{1/2} du = \frac{1}{3}\ u^{3/2}\Big]_{2a^2}^{2a^2} = 0.$
Of course you can see this from the symmetry also.

78. Let $u = x^2$, so $du = 2x\,dx$, and $\int_0^1 x\sqrt{1 - x^4}\, dx = \frac{1}{2}\int_0^1 \sqrt{1 - u^2}\, du$.
This can be interpreted as half the area in the first quadrant inside the unit circle, $\frac{\pi}{8}$.

80. The period from the beginning of the third week to the end of the fourth week is from the end of the second week to the beginning of the fourth week
$\int_2^4 5000\left(1 - \frac{100}{(t + 10)^2}\right) dt = 5000\left(t + \frac{100}{t + 10}\right)\Big]_2^4 = \frac{85000}{21}$.
About 4048 calculators are produced.

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8. The curves $y = x^2$ and $y = x^4$ meet at $(-1, 1)$, $(0, 0)$, and $(1, 1)$ with $y = x^2$ above $y = x^4$ both when $-1 < x < 0$ and when $0 < x < 1$.
The area between the curves is
$\int_{-1}^{1} (x^2 - x^4)\, dx = \left(\frac{1}{3} x^3 - \frac{1}{5} x^5\right)\Big]_{-1}^{1} = \frac{4}{15}$.
See graph to the right.