RevEng

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A Cautionary Example:

$$\sqrt{\pi} \doteq \frac{1}{10^5} \sum_{n=-\infty}^{\infty} e^{-n^2/10^{10}}$$

This is correct to over 42 billion digits but not to 43 billion digits.

- Coincidence?

- “Messing” around only works if you know where to look.

Inverse Symbolic Calculation

- What is 1.198140234735592207?

If $a_0 := 1, b_0 := \sqrt{2}$ and

$$a_{n+1} := \frac{a_n + b_n}{2}, \quad b_{n+1} := \sqrt{a_n b_n}$$

Then

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = \frac{\pi/2}{\int_0^1 \frac{dt}{\sqrt{1-t^4}}}$$

$$= 1.198140234735592207...$$

In 1799, Gauss observed this purely numerically and wrote that this result

"will surely open a whole new field of analysis."

- What is 3.14159265358979323856?

- What is 2.71828182845904523536?

- What is 14.86134161768747018626?

- What is $(e^{\pi\sqrt{163}} - 744)^{\frac{1}{3}}$?

$(e^{\pi\sqrt{163}} - 744)^{\frac{1}{3}} = 640320$ to 30 digits

- However to thirty five places it equals

640319.99999999999999999999999939032

```
> ID(14.86134161768747O186);

  Input is not a rational of small height.

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  Input is not a small height algebraic
  number of degree between 2 and 6.

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  Input is not an elementary function of
  a small rational.
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Input is not an elementary function of a small height algebraic number.

No relation detected between input and selected transcendentals.

Input matches: 3*Pi+2*exp(1)

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> ID(2.856463132680503);

  Input is not a rational of small height.

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  Input matches the following radical
     : sqrt( 2 )  +  ( 3 )^(1/3)

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